**College Algebra — Joysheet 9**
**MAT 140, Fall 2016 — D. Ivanšić**

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*Covers: 5.4–5.6* *Show all your work!*

**1.** (5pts) If $\log_a 4 = 0.548869$ and $\log_a 7 = 0.770435$, find (show how you obtained your numbers):

$\log_a 28 = \log_a(4 \cdot 7)$
$= \log_a 4 + \log_a 7$
$= 0.548869 + 0.770435$
$= 1.319304$

$\log_a \frac{4}{49} = \log_a 4 - \log_a 49$ ($7^2$)
$= \log_a 4 - 2\log_a 7$
$= 0.548869 - 2 \cdot 0.770435$
$= -0.992001$

**2.** (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_6(216x^4y^7) = \log_6 216 + \log_6 x^4 + \log_6 y^7$
$= 3 + 4\log_6 x + 7\log_6 y$

$\log \frac{1000x^2y^{\frac{3}{2}}}{\sqrt{x^5y}} = \log(1000x^2y^{\frac{3}{2}}) - \log(x^5y)^{\frac{1}{2}}$
$= \log 1000 + \log x^2 + \log y^{\frac{3}{2}} - \frac{1}{2}(\log x^5 + \log y)$
$= 3 + 2\log x + \frac{3}{2}\log y - \frac{1}{2}(5\log x + \log y) = 3 - \frac{1}{2}\log x + \log y$

$2 - \frac{5}{2}$ $\qquad$ $\frac{3}{2} - \frac{1}{2} = 1$

**3.** (12pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{4}\ln(16x^8) - 3\ln(2y^{\frac{5}{6}}) - 2\ln x = \ln(16x^8)^{\frac{1}{4}} - \ln(2y^{\frac{5}{6}})^3 - \ln(x^2)$

$= \ln \frac{16^{\frac{1}{4}}(x^8)^{\frac{1}{4}}}{2^3(y^{\frac{5}{6}})^3 x^2} = \ln \frac{\sqrt[4]{16}\, x^2}{8y^{\frac{5}{2}}x^2} = \ln \frac{\cancel{2}\,1}{\cancel{8}\,4\, y^{\frac{5}{2}}} = \ln \frac{1}{4y^{\frac{5}{2}}}$

$3\log_7(x^2 - 12x + 32) - 2\log_7(x-4) - \log_7(x-8) =$

$= \log_7(x^2 - 12x + 32)^3 - \log_7(x-4)^2 - \log_7(x-8)$

$= \log_7 \frac{(x^2-12x+32)^3}{(x-4)^2(x-8)} = \log_7 \frac{(x-4)^{\cancel{3}\,1}(x-8)^{\cancel{3}\,2}}{(x-4)^{\cancel{2}}\cancel{(x-8)}} = \log_7((x-4)(x-8)^2)$

$(x-4)(x-8)$

Solve the equations.

**4.** (5pts) $27^{4x+5} = \left(\frac{1}{9}\right)^{1-2x}$

$$(3^3)^{4x+5} = (3^{-2})^{1-2x}$$

$$3^{12x+15} = 3^{-2+4x}$$

$$12x+15 = -2+4x \quad \Big| -4x-15$$

$$8x = -17$$

$$x = -\frac{17}{8}$$

**5.** (7pts) $6^{4x+1} = 2^{x-5}$ $\quad | \ln$

$$\ln 6^{4x+1} = \ln 2^{x-5}$$

$$(4x+1)\ln 6 = (x-5)\ln 2$$

$$4\ln 6 \cdot x + \ln 6 = \ln 2 \cdot x - 5\ln 2 \quad \Big| \begin{array}{l} -\ln 2 \cdot x \\ -\ln 6 \end{array}$$

$$4\ln 6 \cdot x - \ln 2 \cdot x = -5\ln 2 - \ln 6$$

$$x(4\ln 6 - \ln 2) = -5\ln 2 - \ln 6$$

$$x = -\frac{5\ln 2 + \ln 6}{4\ln 6 - \ln 2} = -0.812108$$

**6.** (8pts) $\log_9(x-5) + \log_9(x+3) = 1$

$$\log_9((x-5)(x+3)) = 1 \quad 9^{\,}$$

$$9^{\log_9((x-5)(x+3))} = 9^1$$

$$(x-5)(x+3) = 9$$

$$x^2 - 2x - 15 = 9$$

$$x^2 - 2x - 24 = 0$$

$$(x-6)(x+4) = 0$$

$x = 6, -4$ – gives negative in log.

$x = 6$ only solution

**7.** (12pts) According to US census data, Indianapolis, IN, had 781,926 inhabitants in 2000 and 820,445 in 2010. Assume the population of Indianapolis grows exponentially.
a) Write the function describing the number $P(t)$ of people in Indianapolis $t$ years after 2000. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 1,000,000?

a) $P(t) = P_0 e^{kt}$

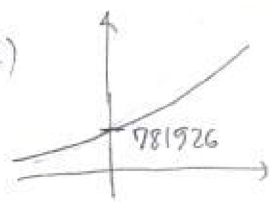
$$820{,}445 = 781{,}926\, e^{k \cdot 10}$$

$$\frac{820{,}445}{781{,}926} = e^{k \cdot 10} \quad | \ln$$

$$\ln \frac{820{,}445}{781{,}926} = k \cdot 10$$

$$k = \frac{\ln \frac{820{,}445}{781{,}926}}{10} = 0.00480868$$

b)




c) $1000000 = 781926\, e^{0.00480868 \cdot t}$ $\quad | \div 781926$

$$1.278\ldots = e^{0.004\ldots t} \quad | \ln$$

$$\ln 1.278\ldots = 0.004\ldots t$$

$$t = \frac{\ln 1.278\ldots}{0.004\ldots} = 51.156519$$

about 51 years from 2000, i.e. in 2051