College Algebra — Joysheet 6
MAT 140, Fall 2016 — D. Ivanšić

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Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $3 + i + (2 - 3i)^2 =$ $3+i+2^2-2\cdot2\cdot3i+(3i)^2$

$= 3+i+4-12i-9 = -2-11i$

2. (6pts) $\frac{4+3i}{5-2i} = \frac{4+3i}{5-2i}\cdot\frac{5+2i}{5+2i} = \frac{20+15i+8i+6i^2}{5^2-(2i)^2} = \frac{20+23i-6}{25-(-4)} = \frac{14+23i}{29}$

3. (4pts) Simplify and justify your answer.

$i^{214} = i^{212}\cdot i^2 = (i^4)^{53}\cdot i^2 = i^2 = -1$

$4\cdot53 = 212$ ($i^4 = 1$, $1^{53} = 1$)

4. (8pts) The number of crates of apples in storage of an apple grower is described by the function $C(x) = -x^2 + 45x + 16$, where x is the number of days after September 1st. On what dates did the apple grower have 390 crates in storage?

$-x^2+45x+16 = 390$

$x^2-45x+390-16=0$

$x^2-45x+374=0$

$x = \frac{-(-45)\pm\sqrt{(-45)^2-4\cdot1\cdot374}}{2\cdot1} = \frac{45\pm\sqrt{2025-1496}}{2}$

$= \frac{45\pm\sqrt{529}}{2} = \frac{45\pm23}{2} = \frac{68}{2}, \frac{22}{2} = 34, 11$

11 days after Sep 1st is Sep. 12th

34 days after Sep 1st is Oct 5th

5. (8pts) Solve the equation: $3x^4 - 7x^2 - 20 = 0$

$3(x^2)^2-7x^2-20=0$

Let $u = x^2$

$3u^2-7u-20=0$

$u = \frac{-(-7)\pm\sqrt{(-7)^2-4\cdot3\cdot(-20)}}{2\cdot3} = \frac{7\pm\sqrt{49+240}}{6} = \frac{7\pm\sqrt{289}}{6} = \frac{7\pm17}{6} = \frac{24}{6}, -\frac{10}{6}$

$u = 4, -\frac{5}{3}$

$x^2 = 4$ $\quad x = \pm2$

$x^2 = -\frac{5}{3}$ $\quad x = \pm\sqrt{\frac{5}{3}}\,i$

6. (6pts) Solve by completing the square.

$x^2 - 16x + 19 = 0$

$+8^2$

$x^2-2\cdot x\cdot8+8^2+19 = 8^2$

$(x-8)^2+19 = 64$ $\quad -19$

$(x-8)^2 = 45$

$x-8 = \pm\sqrt{45}$ (9.5)

$x-8 = \pm3\sqrt{5}$

$x = 8\pm3\sqrt{5}$

7. (12pts) The quadratic function $f(x) = -2x^2 + 5x - 4$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) y-int: $f(0) = -4$

x-int: $-2x^2 + 5x - 4 = 0$

$2x^2 - 5x + 4 = 0$

$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$

$= \frac{5 \pm \sqrt{25 - 32}}{4} = \frac{5 \pm \sqrt{-7}}{4}$

no real sol, so no x-int

b) $h = -\frac{b}{2a} = -\frac{5}{2 \cdot (-2)} = \frac{5}{4}$

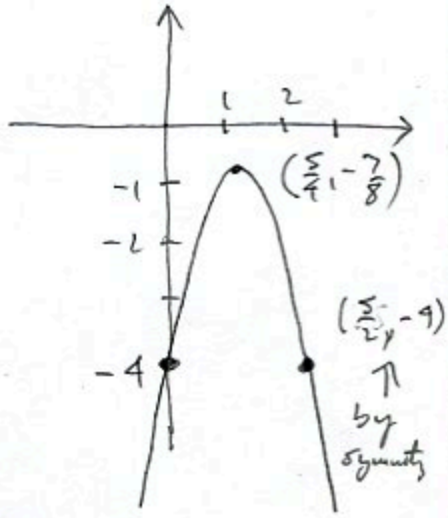
$k = f\left(\frac{5}{4}\right) = -2\left(\frac{5}{4}\right)^2 + 5 \cdot \frac{5}{4} - 4$

$= -2 \cdot \frac{25}{16} + \frac{25}{4} - 4$

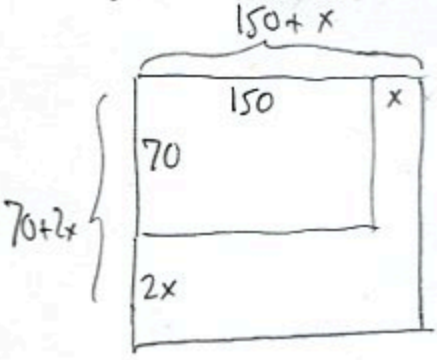
$= -\frac{25}{8} + \frac{25}{4} - 4$

$= \frac{-25 + 50 - 32}{8}$

$= -\frac{7}{8}$ $\left(\frac{5}{4}, -\frac{7}{8}\right)$



8. (12pts) Donald's house sits on a big rectangular plot of land that is 150 by 70 yards. He wishes to enlarge it to get a rectangular plot with area 20,000 square yards by extending the 150-yard side by a certain amount and increasing the 70-yard side by twice that amount. By how much should the 150- and 70-yard sides be extended to achieve the desired area?



x = amount the 150-yard side is enlarged

$(70 + 2x)(150 + x) = 20000$

$10500 + 70x + 300x + 2x^2 = 20000 \quad | -20000$

$2x^2 + 370x - 9500 = 0 \quad | \div 2$

$x^2 + 185x - 4750 = 0$

$x = \frac{-185 \pm \sqrt{185^2 - 4 \cdot 1 \cdot (-4750)}}{2 \cdot 1} = \frac{-185 \pm \sqrt{53225}}{2}$

$25.\overline{2129}$

$= \frac{-185 \pm 5\sqrt{2129}}{2}$ $\quad \frac{-185 - 5\sqrt{2129}}{2} < 0$, so is not a sol., since $x \geq 0$

150 yard side lengthened by $\frac{-185 + 5\sqrt{2129}}{2} = 22.85272$

70 yard side lengthened by $-185 + 5\sqrt{2129} = 45.70544$ yds