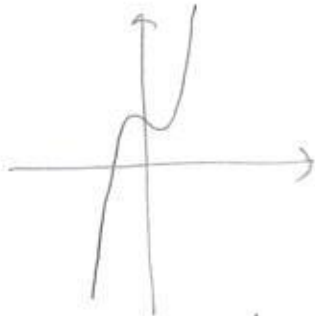**1.** (21pts) For the following functions:
a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$f(x) = x^3 - x + 4$

$g(x) = x^5 - 13x^3 + 36x$

$h(x) = x^2 - 4|x| - 2$

$f(-x) = (-x)^3 - (-x) + 4$

$= -x^3 + x + 4 \quad \begin{array}{l}\ne f(x)\\ \ne -f(x)\end{array}$

neither

$g(-x) = (-x)^5 - 13(-x)^3 + 36(-x)$

$= -x^5 - 13(-x^3) + 36(-x)$

$= -x^5 + 13x^3 - 36x$

$= -g(x) \quad$ odd

$h(-x) = (-x)^2 - 4|-x| - 2$

$= x^2 - 4|x| - 2$

$= h(x)$

even

no symmetry,

so neither

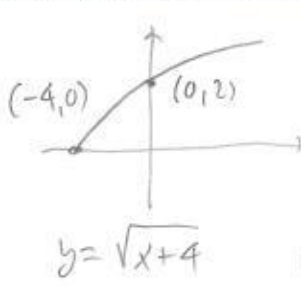symmetric about
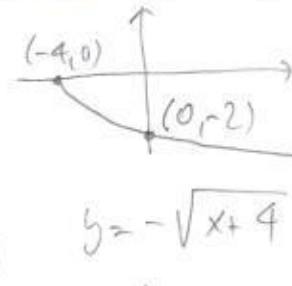
origin, so odd
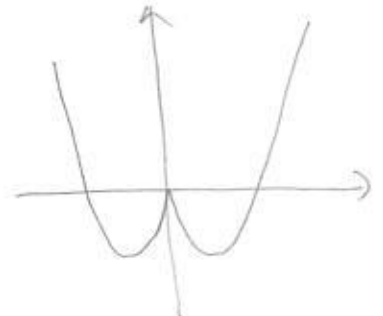
symmetric about y-axis,

so even

2. (16pts) Using transformations, draw the graphs of $f(x) = -\sqrt{x+4}$ and $g(x) = \dfrac{2}{-x} + 5$.
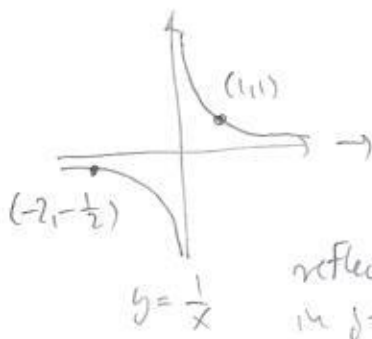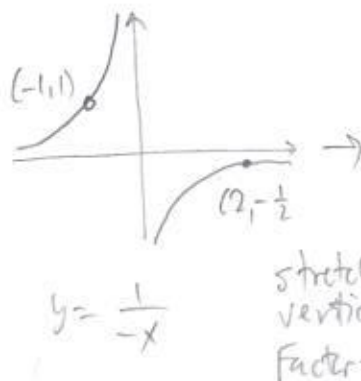Explain how you transform graphs of basic functions in order to get the graphs of f and g.
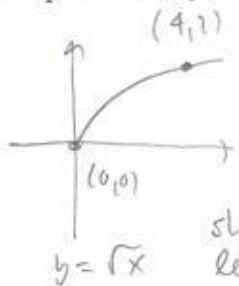
$(4,2)$

$(0,0)$

$y = \sqrt{x}$
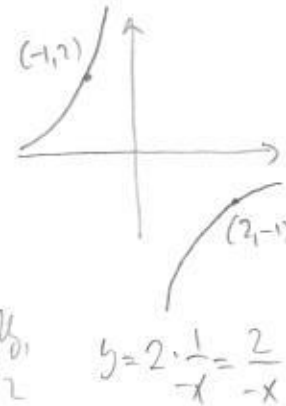
shift
left 4

$(-4,0)$ $(0,2)$

$y = \sqrt{x+4}$

reflect
in x-axis

$(-4,0)$

$(0,-2)$

$y = -\sqrt{x+4}$

$(-1,7)$

$(2,4)$

$(1,1)$

$(-2,-\tfrac{1}{2})$

$y = \dfrac{1}{x}$

reflect
in y-axis

$(-1,1)$

$(2,-\tfrac{1}{2})$
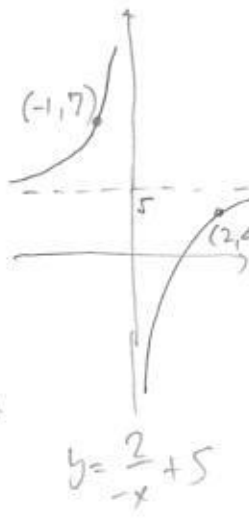
$y = \dfrac{1}{-x}$

stretch
vertically,
factor 2

$(-1,2)$

$(2,-1)$

$y = 2 \cdot \dfrac{1}{-x} = \dfrac{2}{-x}$

shift
up
5

$y = \dfrac{2}{-x} + 5$

3. (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^3$, shifted right 4 units
b) shape of $y = \frac{1}{x}$ stretched horizontally by factor 4, then shifted up 3 units
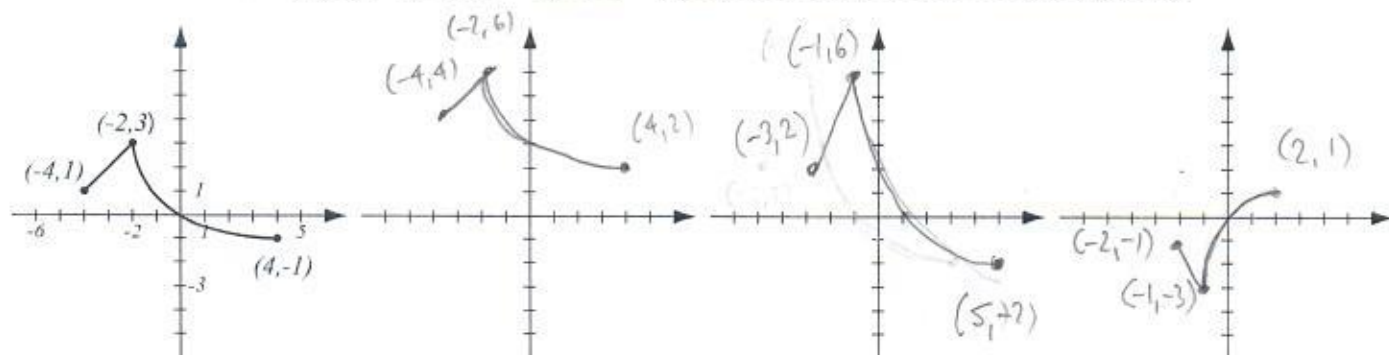c) shape of $y = |x|$, stretched vertically by factor 2, then reflected about the x-axis, then shifted down 1 unit.

a) $y = (x - 4)^3$

b) $y = \frac{1}{\frac{1}{4}x} + 3 = \frac{4}{x} + 3$

c) $y = -2|x| - 1$

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x) + 3$, $2f(x-1)$ and $-f(2x)$ and label all the relevant points.

$f(x) + 3$

shift up 3

$2f(x-1)$

shift right 1,
stretch vertically,
factor = 2

$-f(2x)$

stretch horizontally,
factor = $\frac{1}{2}$,
reflect in y-axis