College Algebra — Joysheet 4
MAT 140, Fall 2016 — D. Ivanšić

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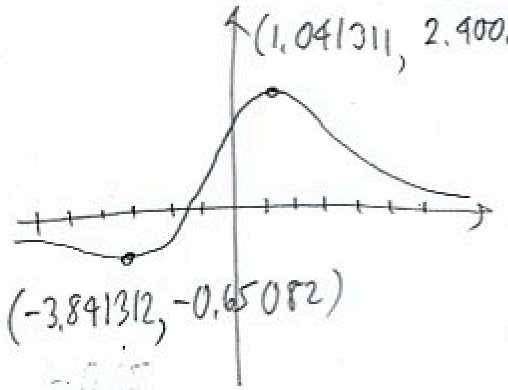
Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{5x+7}{x^2+4}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.



a) $f(1.041311) = 2.40082$ is a local max

$f(-3.841312) = -0.65082$ is a local min

b) Increasing on $(-3.841312, 1.041311)$

Decreasing on $(-\infty, -3.841312)$ and $(1.041311, \infty)$

2. (20pts) Let $f(x) = 3\sqrt{x} - 1$, $g(x) = \dfrac{2x-1}{x^2+x-6}$. Find the following (simplify where possible):

$(f+g)(4) = f(4) + g(4) = 3\sqrt{4} - 1 + \dfrac{2\cdot 4-1}{4^2+4-6}$

$= 5 + \dfrac{7}{14} = 5.5 = \dfrac{11}{2}$

$(fg)(8) = f(8)\cdot g(8) = (3\sqrt{8}-1)\cdot \dfrac{2\cdot 8-1}{8^2+8-6}$

$= (3\sqrt{8}-1)\dfrac{15}{66} = \dfrac{5(3\sqrt{8}-1)}{22}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{3\sqrt{x}-1}{\frac{2x-1}{x^2+x-6}} =$

$= \dfrac{3\sqrt{x}-1}{1} \cdot \dfrac{x^2+x-6}{2x-1} = \dfrac{(3\sqrt{x}-1)(x^2+x-6)}{2x-1}$

$(g\circ f)(9) = g(f(9)) = g(3\cdot\sqrt{9}-1)$

$= g(8) = \dfrac{2\cdot 8-1}{8^2+8-6} = \dfrac{15}{66} = \dfrac{5}{22}$

$(f\circ g)(x) = f(g(x)) = f\left(\dfrac{2x-1}{x^2+x-6}\right) = 3\sqrt{\dfrac{2x-1}{x^2+x-6}} - 1$

The domain of $\dfrac{f}{g}(x)$ in interval notation

domain of f must have $x \geq 0$

Domain of g: can't have: $x^2+x-6=0$ $x=-3,2$
$(x+3)(x-2)=0$

f — 0
g — -3 2
$\frac{f}{g}$ — overlap & delete $\frac{1}{2}$

Can't have: $g(x)=0$ so $2x-1=0$ $x=\frac{1}{2}$

Domain: $[0,\frac{1}{2})\cup(\frac{1}{2},2)\cup(2,\infty)$

3. (8pts) Consider the function $h(x) = (4x-7)^2$ and find two different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = 4x-7$ $\qquad$ $g(x) = 4x$

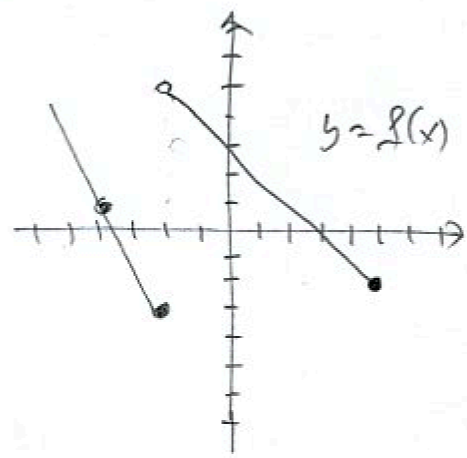
$f(x) = x^2$ $\qquad$ $f(x) = (x-7)^2$

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -2x-7, & \text{if } x \le -2 \\ 3-x, & \text{if } -2 < x \le 5. \end{cases}$$

x	-2x-7
-2	-3
-4	1

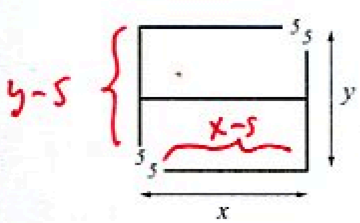
x	3-x
-2	5
5	-2



5. (14pts) A retail chain with two brands is opening two new stores, to be housed in the same building with total area 4000 square feet. The two stores share a wall and have 5-ft openings on the corners to allow for entrances (see picture). The chain wishes to minimize the total length of the walls.

a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the building for which the total length of the walls is minimal? What is the minimal wall length?



a) $l = x + 2(x-5) + 2(y-5)$

$= 3x + 2y - 20$

$4000 = xy$, so $y = \frac{4000}{x}$

$l = 3x + 2 \cdot \frac{4000}{x} - 20$

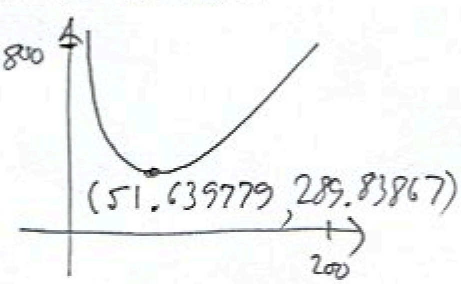
$l(x) = 3x + \frac{8000}{x} - 20$

Domain: Must have: $x \geq 5$ $\qquad$ $y \geq 5$ $\qquad$ $4000 \geq 5x$

so $\qquad$ $\frac{4000}{x} \geq 5$ $\qquad$ $800 \geq x$

$[5, 800]$

b)



Dimensions: $\qquad$ 4000/51.63...

51.639779 × 77.459665 ft

Min length of walls is

289.83867 ft