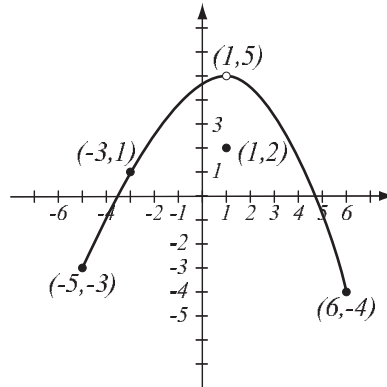


1. (8pts) Use the graph of the function f at right to answer the following questions.

- Find: $f(-3) =$ $f(1) =$
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = -2$?



2. (10pts) Use your calculator to accurately sketch the graph of $y = -x^3 + 3x^2 + 5x + 7$. Draw the graph here, and indicate units on the axes. Find all the x - and y -intercepts (accuracy: 6 decimal points).

3. (5pts) Write the equation of the line whose slope is -2 and whose x -intercept is 3 .

4. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through point $(1, -3)$ and is perpendicular to the line $3x + 2y = 4$. Draw both lines.

5. (8pts) Draw the triangle with vertices $A = (-3, -1)$, $B = (6, 4)$ and $C = (1, 7)$. Use either slopes or distance to determine if this is a right triangle.

6. (9pts) Let $g(x) = \frac{x-1}{x^2-3}$. Find the following (simplify where appropriate).

$$g(-4) = \qquad \qquad \qquad g(\sqrt{3}) =$$

$$g(-2x) = \qquad \qquad \qquad g(a+2) =$$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$$f(x) = \frac{x-13}{x^2+3x-18}$$

$$g(x) = \sqrt{2x-5}$$

8. (5pts) Solve the inequality and write your solution in interval notation.

$$2 \leq 3x - 2 \leq 7$$

9. (10pts) A circle has center $(-2, 3)$ and passes through the point $(1, 2)$.

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

10. (12pts) Frank, a salesman at a clothing store, can be paid under one of two plans:

A) salary of \$1,250 plus 10% of sales over \$1,000

B) salary of \$1,100 plus 20% of sales over \$2,000.

Assuming Frank always has sales of at least \$2,000, for which amount of sales is plan A better?

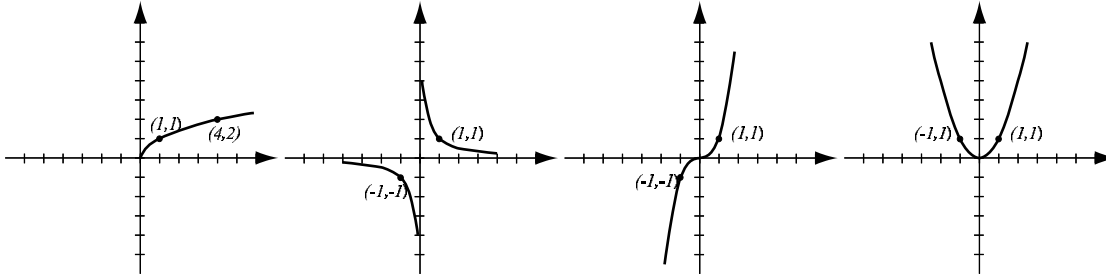
11. (14pts) A truck starts driving eastward from Murray along state route 80. A car driving 8mph faster starts along the same route half an hour afterwards. After the car drives three and a half hours, it catches up with the truck.

a) How fast are the truck and the car?

b) How far from Murray are they when the car catches up with the truck?

Bonus (10pts) Two containers with 40% and 65% solutions of orthophosphoric acid were mixed. The result was 5 liters of a 47% solution of orthophosphoric acid. How much acid solution was in each of the two containers?

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let $f(x) = \frac{x^2}{x-7}$, $g(x) = \sqrt{2x+9}$.

Find the following (simplify where possible):

$$(f - g)(8) =$$

$$(fg)(1) =$$

$$\frac{g}{f}(x) =$$

$$(g \circ f)(9) =$$

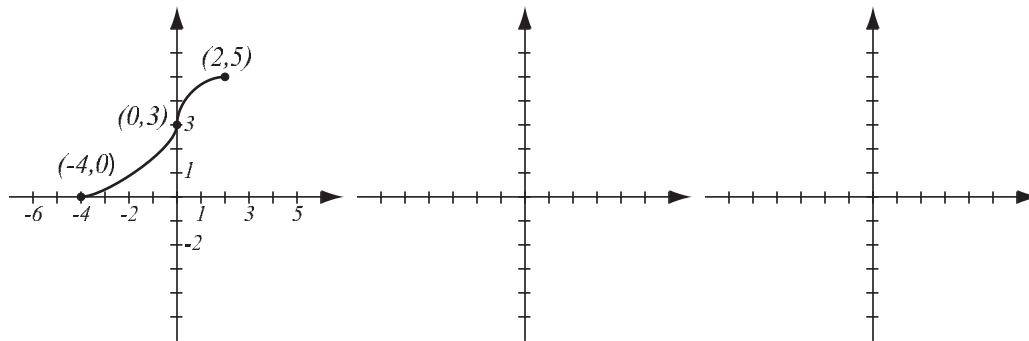
$$(f \circ g)(x) =$$

The domain of $f + g$ in interval notation

3. (6pts) Consider the function $h(x) = \sqrt[3]{5x+2}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (8pts) Using transformations, draw the graph of $f(x) = 2|x| + 1$. Explain how you transform the graph of a basic function (which one?) in order to get the graph of f .

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x-2) - 3$ and $f(-2x)$ and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x - 4, & \text{if } -3 < x < 2 \\ -3x + 5, & \text{if } x \geq 2 \end{cases}$$

7. (8pts) Simplify.

$$\frac{2x}{x^2 + 2x - 3} - \frac{x - 3}{x^2 + 4x - 5} =$$

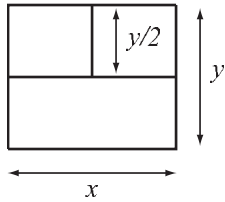
8. (18pts) Let $f(x) = -x^3 + 8x$ (answer with 6 decimal points accuracy).

- a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
- b) Determine algebraically whether the function is odd, even, or neither.
- c) Verify your conclusion from b) by stating symmetry.
- d) Find the local maxima and minima for this function.
- e) State the intervals where the function is increasing and where it is decreasing.

9. (14pts) Georgina is planning a simple 3-room house with an area of 900 square feet (see picture). She wishes to minimize the total length of the walls.

a) Express the total length of the walls as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the length function here and find its minimum. What are the dimensions of the house for which the total length of the walls is minimal? What is the minimal wall length?



Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this to find the point on the curve $y = x^2$ that is closest to the point $(1, 2)$. *Hint: minimize the distance from a point (x, y) on the curve to the point $(1, 2)$. Make it a function only of x .*

College Algebra — Exam 3
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Name: _____
Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $(1 + i)^3 =$

2. (5pts) $\frac{2 + 5i}{4 - i} =$

3. (4pts) Simplify and justify your answer.

$i^{91} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 10x + 33 = 0$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|2x - 5| < 7$

6. (6pts) Let $P(x)$ be a polynomial.

- If the graph of P has 3 x -intercepts, what can you say about the degree of P ?
- If the graph of P has 5 turning points, what can you say about the degree of P ?
- Can the graph of P have 4 x -intercepts and 2 turning points? Explain why or why not.

7. (12pts) The quadratic function $f(x) = x^2 + 3x - 18$ is given. Do the following without using the calculator.

a) Find the x - and y -intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

Solve the equations:

8. (8pts) $1 - \frac{6}{x+1} = \frac{4x+16}{x^2+5x+4}$

9. (8pts) $x = \sqrt{x+9} + 3$

10. (14pts) The polynomial $f(x) = -(x + 3)^2(x - 4)(x - 5)$ is given.

a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the y -intercept.

c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).

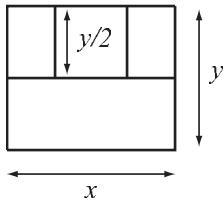
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) In a rectangle, the sides are 3 ft and 6ft shorter than the diagonal. What are the dimensions of the rectangle?

12. (14pts) A company is building a conference center whose floorplan is below. It has budgeted enough money to build 750 feet of walls, and its goal is to maximize the total area of the building.

a) Express the total area of the building as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Solve the following equation for a complex number z . Your answer should be in form $a + bi$. *Hint: work like you would with real numbers to isolate z .*

$$4 + iz = 2z - 3i$$

College Algebra — Exam 4
MAT 140, Fall 2016 — D. Ivanišić

Name: _____
Show all your work!

1. (8pts) Evaluate without using the calculator:

$$\log_9 729 = \quad \log_2 \frac{1}{8} = \quad \log_c \sqrt[7]{c^3} = \quad \log_{\sqrt{b}} b^4 =$$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_3 0.13$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 5 = u$ and $\log_a 9 = v$, express in terms of u and v :

$$\log_a \frac{9}{5} = \quad \log_a 15 =$$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 \frac{x^4}{9\sqrt{y^7}} =$$

5. (6pts) Write as a single logarithm. Simplify if possible.

$$3\log_2(x^{-2}y^4) - 4\log_2(x^2y^5) =$$

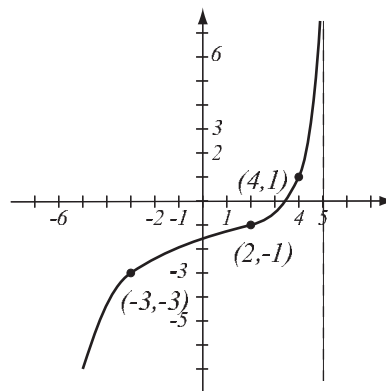
6. (4pts) Simplify.

$$\log 10^{x-3} = \quad 4^{\log_4(7x)} =$$

7. (6pts) The graph of a function f is given.

a) Is this function one-to-one? Justify.

b) If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points, and showing any asymptotes.



8. (9pts) Let $f(x) = \frac{4x - 2}{2x + 3}$.

a) Find the formula for f^{-1} .

b) Find the range of f^{-1} .

9. (6pts) Using transformations, draw the graph of $f(x) = 2 - 3^x$. Explain how you transform the graph of a basic function in order to get the graph of f . Indicate at least one point on the graph and any asymptotes.

10. (6pts) Find the domain of the function $f(x) = \frac{\log_3(2x - 7)}{\log_7(5 - x)}$ and write it in interval notation.

11. (8pts) How much should you invest in an account bearing 3.1%, compounded quarterly, if you wish to have \$1,000 in five years?

Solve the equations.

12. (6pts) $16^{3x-2} = \left(\frac{1}{8}\right)^{x+1}$

13. (4pts) $5^{2x} = 4$

14. (10pts) $2^{2x} - 16 = 6 \cdot 2^x$

15. (12pts) The population of Fecund Grove was 14,000 in 2005 and 22,000 in 2011. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.

a) Find k and write the function that describes the population at time t years since 2005. Graph it on paper.

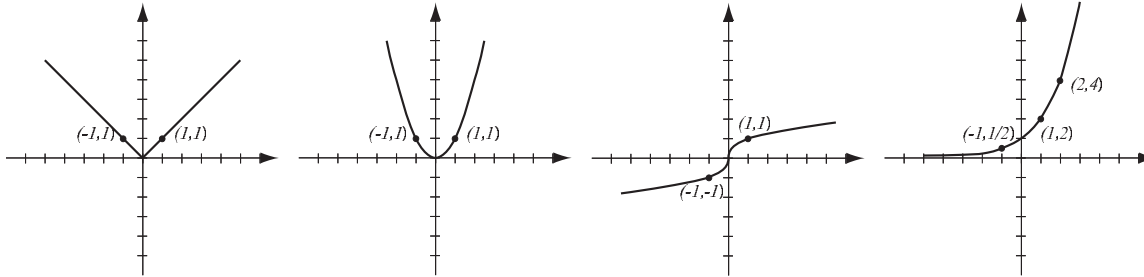
b) Find the predicted population in the year 2021.

Bonus (10pts) Let $f(x) = \frac{3}{1 + e^{-x}}$.

a) Find the inverse function of f .

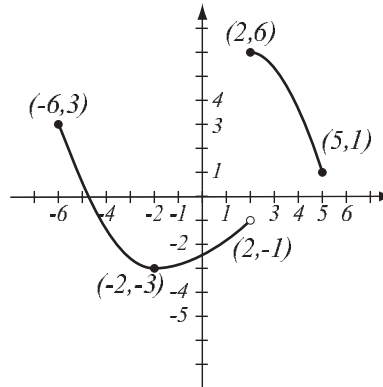
b) Show that $f^{-1}(f(x)) = x$.

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (8pts) Use the graph of the function f at right to answer the following questions.

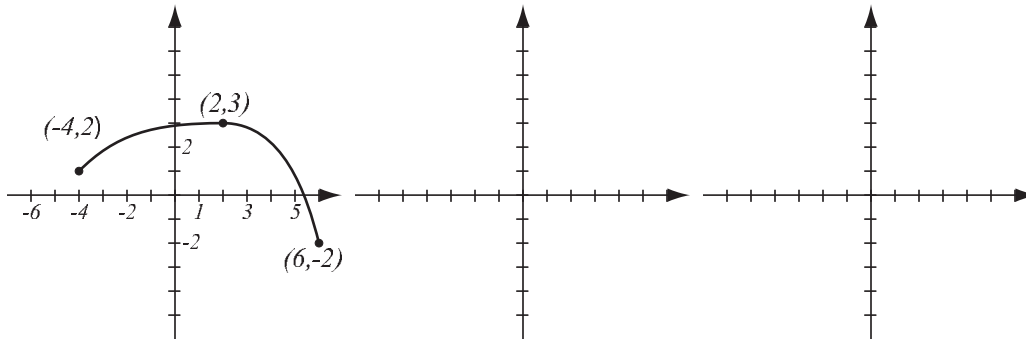
- Find: $f(-2) =$ $f(2) =$
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = 3$?



3. (5pts) Write the equation of the line that passes through points $(1, 3)$ and $(-2, 2)$.

4. (9pts) Find the equation of the line (in form $y = mx + b$) that passes through point $(4, -1)$ and is perpendicular to the line $5x - 2y = 8$. Draw both lines.

5. (10pts) The graph of $f(x)$ is drawn below. Draw the graphs of $3f(x + 2)$ and $-f(2x)$ and label all the relevant points.



6. (12pts) The quadratic function $f(x) = x^2 - 4x + 7$ is given. Do the following without using the calculator.
- Find the x - and y -intercepts of its graph, if any.
 - Find the vertex of the graph.
 - Sketch the graph of the function.

7. (6pts) Solve the inequality. Write the solution in interval form.

$$|x + 7| \geq 9$$

8. (6pts) Let $f(x) = \frac{2x}{2x - 5}$. Find the formula for f^{-1} .

9. (6pts) Write as a single logarithm. Simplify if possible.

$$2 \log_5(x^4 y^{-3}) - 3 \log_5(xy^5) =$$

10. (6pts) Find the domain of the function $f(x) = \frac{\ln(3x - 6)}{\sqrt{9 - 2x}}$ and write it in interval notation.

- 11.** (20pts) The polynomial $P(x) = x^4 - 9x^2$ is given (answer with 6 decimals accuracy).
- What is the end behavior of the polynomial?
 - Factor the polynomial to find all the zeros and their multiplicities. Find the y -intercept.
 - Determine algebraically whether the function is odd, even, or neither.
 - Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
 - Verify your conclusion from c) by stating symmetry.
 - Find all the turning points (i.e., local maxima and minima).

Solve the equations.

12. (6pts) $8^{3x-4} = \left(\frac{1}{16}\right)^{x+4}$

13. (8pts) $x = 3 + \sqrt{37 - 3x}$

14. (14pts) A truck starts driving eastward from Murray along state route 80. A car driving 11mph faster starts along the same route half an hour afterwards. After the car drives two and a half hours, it catches up with the truck.

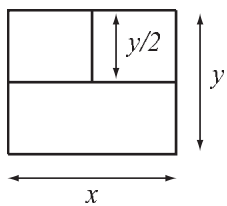
a) How fast are the truck and the car?

b) How far from Murray are they when the car catches up with the truck?

15. (14pts) Georgina is planning a simple 3-room house with an area of 1200 square feet (see picture). She wishes to minimize the total length of the walls.

a) Express the total length of the walls as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the length function here and find its minimum. What are the dimensions of the house for which the total length of the walls is minimal? What is the minimal wall length?



16. (12pts) The population of Fecund Grove was 17,000 in 2001 and 31,000 in 2009. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.

a) Find k and write the function that describes the population at time t years since 2001. Graph it on paper.

b) Find the predicted population in the year 2018.

Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this to find the point on the curve $y = x^2$ that is closest to the point $(1, 2)$. *Hint: minimize the distance from a point (x, y) on the curve to the point $(1, 2)$. Make it a function only of x .*