College Algebra — Final Exam MAT 140, Spring 2016 — D. Ivanšić Name:

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (8pts) Use the graph of the function f at right to answer the following questions.

- a) What is the domain of f?
- b) What is the range of f?
- c) Find the intervals of increase of f.
- d) What are the solutions of the equation f(x) = -4?



3. (6pts) Solve the inequality. Write the solution in interval form.



4. (10pts) Find the equation of the line (in form y = mx + b) whose x-intercept is 3 and passes through (-3, 4). Is this line perpendicular to the line 3x - 2y = 7? Draw both lines.

5. (10pts) The graph of f(x) is drawn below. Find the graphs of -f(x) + 1 and f(-2x) and label all the relevant points.



6. (12pts) The quadratic function $f(x) = x^2 - 6x + 10$ is given. Do the following without using the calculator.

- a) Find the x- and y-intercepts of its graph, if any.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

7. (8pts) Find the domain of the function $f(x) = \frac{\ln(3x-10)}{x^2 - 4x - 12}$ and write it in interval notation.

8. (6pts) Let $f(x) = \frac{2x-3}{x+1}$. Find the formula for f^{-1} .

9. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_2 \frac{x^{-5}}{32\sqrt[4]{y^7}} =$$

10. (6pts) If you invest \$4,000 in an account bearing 3.05%, compounded quarterly, how much do you have in 10 years?

11. (14pts) The polynomial $f(x) = (x-3)(x+2)^2$ is given.

- a) What is the end behavior of the polynomial?
- b) List all the zeros and their multiplicities. Find the *y*-intercept.
- c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
- d) Find all the turning points (i.e., local maxima and minima).

Solve the equations.

12. (8pts) $\sqrt{3x+13}+5=x$

13. (8pts) $4^{x-1} = 14^{3x-5}$

14. (14pts) Greg drove one and a half hours from Paducah to Hopkinsville on a scenic route. On his way back, he took a route 9 miles shorter, so the trip lasted one hour and fifteen minutes. He drove at the same speed on both trips.

a) What was Greg's speed on both trips?

b) How long is the scenic route?

15. (14pts) A developer has budgeted enough money to build 1500 feet of walls in a small retail building consisting of two stores with back rooms, where the front of the store is not walled in (see picture). The developer's goal is to maximize the total area of the building. a) Express the area of the building as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible area?



16. (12pts) The population of Spiriton was 72,000 in 1995 and 94,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.

a) Find k and write the function that describes the population at time t years since 1995. Graph it on paper.

b) Find the predicted population in the year 2017.

Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Consider the set of points (x, y) that has equal distance to the point (0, 2) and the x-axis (distance from a point to the x-axis is the y-coordinate of the point). Write the equation that a point (x, y) with this property has to satisfy, and simplify it to form y = f(x). You should get a parabola. (This is the classical definition of a parabola: set of points whose distance to a fixed point and a fixed line is equal.)

