College Algebra — Final Exam
MAT 140, Fall 2016 — D. Ivanšić

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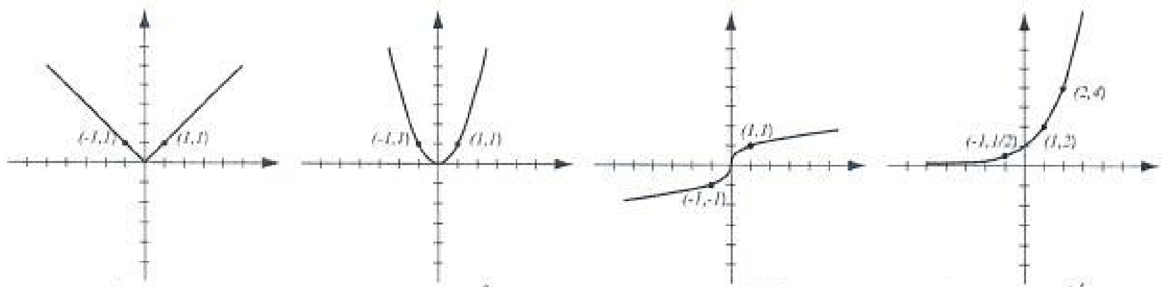
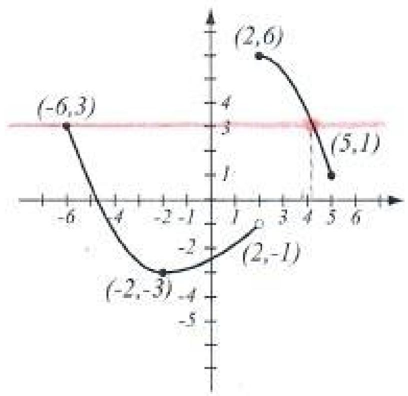
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = |x|$ $\qquad$ $y = x^2$ $\qquad$ $y = \sqrt[3]{x}$ $\qquad$ $y = 2^x$

2. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(-2) =$ 3 $\quad f(2) =$ 6

b) What is the domain of f? $[-6, 5]$

c) What is the range of f? $[-3, 6]$

d) What are the solutions of the equation $f(x) = 3$?

$x = -6, 4$

3. (5pts) Write the equation of the line that passes through points $(1, 3)$ and $(-2, 2)$.

$m = \frac{2-3}{-2-1} = \frac{-1}{-3} = \frac{1}{3}$ $\qquad$ $y - 3 = \frac{1}{3}(x-1)$ $\qquad$ $y = \frac{1}{3}x + \frac{8}{3}$

$y = \frac{1}{3}x - \frac{1}{3} + 3$

4. (9pts) Find the equation of the line (in form $y = mx + b$) that passes through point $(4, -1)$ and is perpendicular to the line $5x - 2y = 8$. Draw both lines.

$5x - 2y = 8$

$2y = 5x - 8 \quad | \div 2$

$y = \frac{5}{2}x - 4$

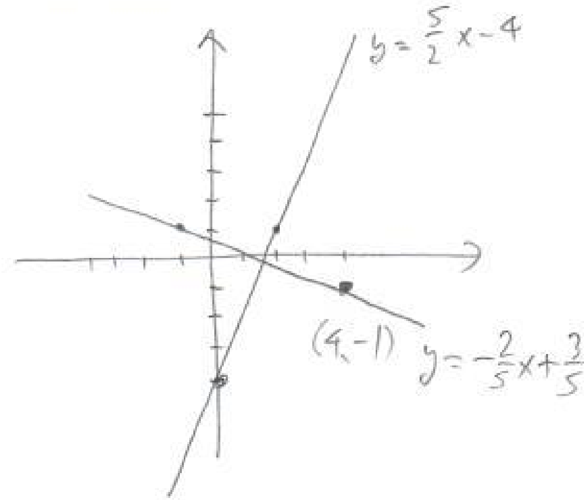
Slope $= \frac{5}{2}$

Our line has slope $-\frac{2}{5}$, passes through $(4, -1)$

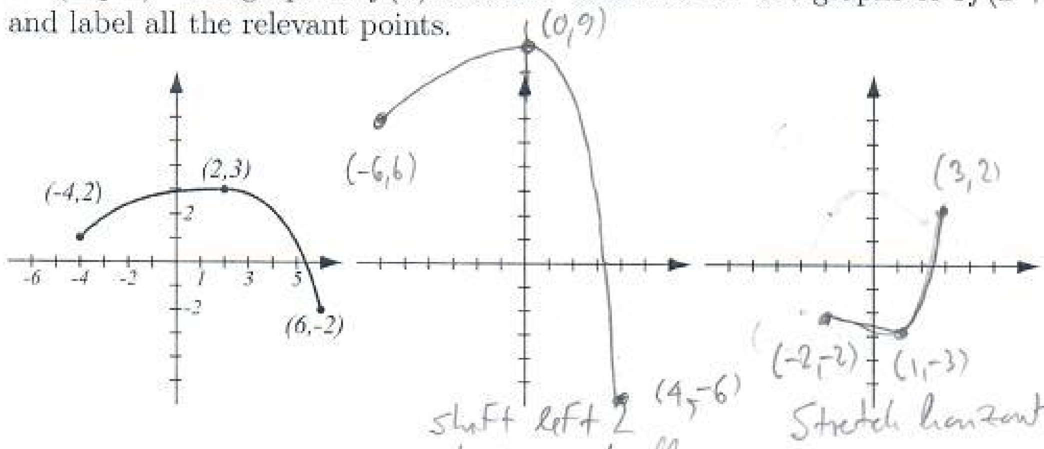
$y - (-1) = -\frac{2}{5}(x - 4)$

$y + 1 = -\frac{2}{5}x + \frac{8}{5}$

$y = -\frac{2}{5}x + \frac{3}{5}$



5. (10pts) The graph of $f(x)$ is drawn below. Draw the graphs of $3f(x+2)$ and $-f(2x)$ and label all the relevant points.



shift left 2
stretch vertically, factor = 3

Stretch horizontally, factor = $\frac{1}{2}$
reflect in x-axis

6. (12pts) The quadratic function $f(x) = x^2 - 4x + 7$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

c) y-int: $f(0) = 7$

x-int: $x^2 - 4x + 7 = 0$

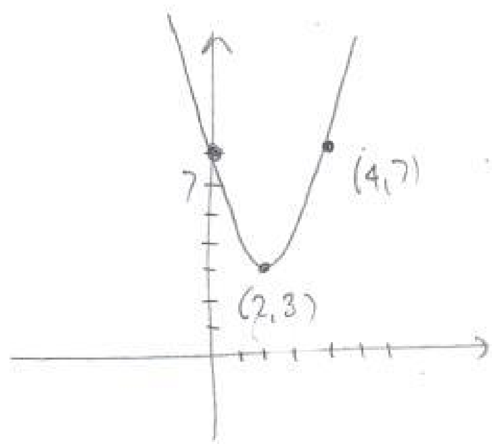
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$$

$$= \frac{4 \pm \sqrt{16-28}}{2} = \frac{4 \pm \sqrt{-12}}{2}$$

no x-int.

b) Vertex: $h = -\frac{-4}{2 \cdot 1} = 2$

$k = f(2) = 4 - 8 + 7 = 3$

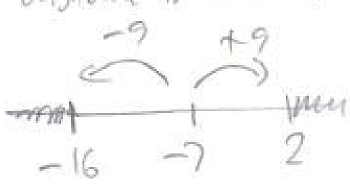


7. (6pts) Solve the inequality. Write the solution in interval form.

$|x+7| \geq 9$

$|x - (-7)| \geq 9$

distance from x to $-7 \geq 9$



$(-\infty, -16] \cup [2, \infty)$

8. (6pts) Let $f(x) = \dfrac{2x}{2x-5}$. Find the formula for f^{-1}.

$$y = \frac{2x}{2x-5}$$

$$(2x-5)y = 2x$$

$$2xy - 5y = 2x$$

$$2xy - 2x = 5y$$

$$x(2y-2) = 5y$$

$$x = \frac{5y}{2y-2}$$

$$f^{-1}(y) = \frac{5y}{2y-2}$$

9. (6pts) Write as a single logarithm. Simplify if possible.

$2\log_5(x^4y^{-3}) - 3\log_5(xy^5) = \log_5(x^4y^{-3})^2 - \log_5(xy^5)^3$

$$= \log_5 \frac{(x^4y^{-3})^2}{(xy^5)^3} = \log_5 \frac{x^8y^{-6}}{x^3y^{15}}$$

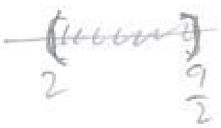
$$= \log_5 x^5y^{-21} = \log_5 \frac{x^5}{y^{21}}$$

10. (6pts) Find the domain of the function $f(x) = \dfrac{\ln(3x-6)}{\sqrt{9-2x}}$ and write it in interval notation.

Must have: $3x-6 > 0$ and $9-2x > 0$

$3x > 6$ $\quad$ $9 > 2x$

$x > 2$ and $x < \frac{9}{2} = 4.5$



$$\left(2, \frac{9}{2}\right)$$

11. (20pts) The polynomial $P(x) = x^4 - 9x^2$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

c) like x^4 $\cup$

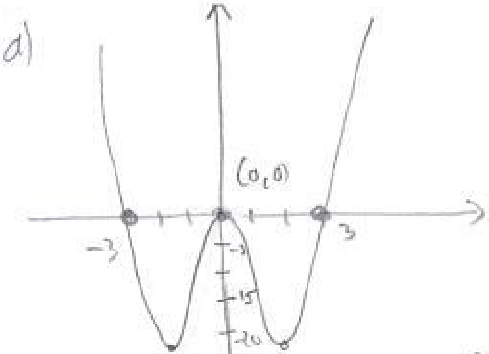
b) $P(x) = x^2(x^2-9) = x^2(x-3)(x+3)$

zero	0	3	-3
mult	2	1	1

$P(0) = 0$

c) $P(-x) = (-x)^4 - 9(-x)^2$
$= x^4 - 9x^2$
$= P(x)$ even

d)



$(-2.121322, -20.25)$ $(2.121322, -20.25)$

e) symmetric with respect to y-axis

f) local max: $f(0) = 0$
local min: $f(-2.121322) = -20.25$
$f(2.121322) = -20.25$

Solve the equations.

12. (6pts) $8^{3x-4} = \left(\frac{1}{16}\right)^{x+4}$

$(2^3)^{3x-4} = (2^{-4})^{x+4}$

$2^{9x-12} = 2^{-4x-16}$

$9x - 12 = -4x - 16$

$13x = -4$

$x = -\frac{4}{13}$

13. (8pts) $x = 3 + \sqrt{37 - 3x}$

$x - 3 = \sqrt{37-3x}$ $\quad |^2$

$x^2 - 6x + 9 = 37 - 3x$ $\quad | +3x - 37$

$x^2 - 3x - 28 = 0$

$(x-7)(x+4) = 0$

$x = 7, -4$

check: $7 = 3 + \sqrt{37-21}$ yes
$-4 = 3 + \sqrt{37+12}$ no

$x = 7$ only solution

14. (14pts) A truck starts driving eastward from Murray along state route 80. A car driving 11mph faster starts along the same route half an hour afterwards. After the car drives two and a half hours, it catches up with the truck.
a) How fast are the truck and the car?
b) How far from Murray are they when the car catches up with the truck?

r= truck's speed
d= distance from Murray when they meet

$d = r \cdot 3$ 2.5+0.5
$d = (r+11)\cdot 2.5$

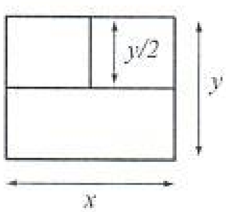
$3r = 2.5r + 27.5 \quad |-2.5r$

$0.5r = 27.5 \quad |\cdot 2$

$r = 55$ mph

b) $3 \cdot 55 = 165$ miles

15. (14pts) Georgina is planning a simple 3-room house with an area of 1200 square feet (see picture). She wishes to minimize the total length of the walls.
a) Express the total length of the walls as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the length function here and find its minimum. What are the dimensions of the house for which the total length of the walls is minimal? What is the minimal wall length?

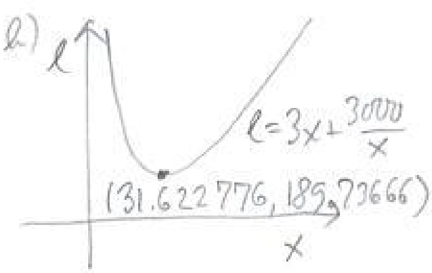


Domain:
Must have $x>0$
$(0,\infty)$

$xy = 1200$, so $y = \frac{1200}{x}$

$\ell = 3x + 2y + \frac{y}{2}$
$= 3x + \frac{5}{2}y$
$= 3x + \frac{5}{2}\cdot\frac{1200}{x}$

$\ell(x) = 3x + \frac{3000}{x}$

b)


Minimal length is for rectangle
31.622776 x 37.947333 ft

Minimal wall length is 189.73666 ft

16. (12pts) The population of Fecund Grove was 17,000 in 2001 and 31,000 in 2009. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2001. Graph it on paper.
b) Find the predicted population in the year 2018.

a) $P(t) = 17e^{kt}$ (in thousands

$31 = 17e^{k8} \quad | \div 17$

$\frac{31}{17} = e^{8k} \quad | \ln$

$\ln\frac{31}{17} = 8k$

$k = \frac{\ln\frac{31}{17}}{8} = 0{,}0750967$

Growth rate is 7.5%

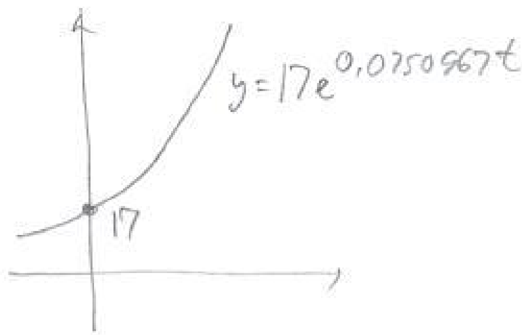
$P(t) = 17 \cdot e^{0{,}0750967}$

b) $t = 17$

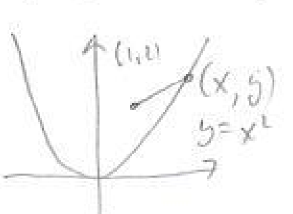
$P(17) = 17 \cdot e^{0{,}0750967 \cdot 17}$

$= 60.938051$

About 60,938 inhabitants

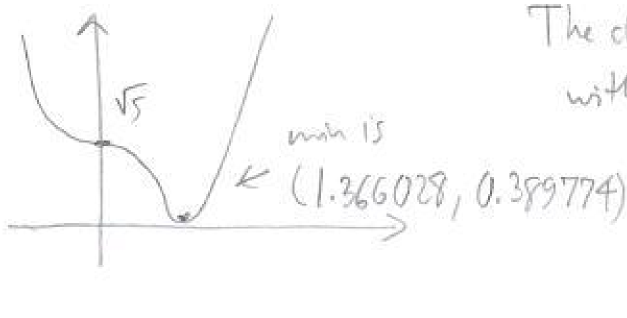


Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this to find the point on the curve $y = x^2$ that is closest to the point $(1, 2)$. *Hint: minimize the distance from a point (x, y) on the curve to the point $(1, 2)$. Make it a function only of x.*



$d = \sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-1)^2 + (x^2-2)^2}$

$= \sqrt{x^2 - 2x + 1 + x^4 - 4x^2 + 4} = \sqrt{x^4 - 3x^2 - 2x + 5} = d(x)$



The closest point is (1,366028, 1,866032) with smallest distance 0,389774