College Algebra — Exam 4
MAT 140, Spring 2016 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_9 729 = 3$ $\quad$ $\log_2 \frac{1}{8} = -3$ $\quad$ $\log_c \sqrt[7]{c^3} = \frac{3}{7}$ $\quad$ $\log_{\sqrt{b}} b^4 = 8$

$9^? = 729$ $\quad$ $2^? = \frac{1}{8} = \frac{1}{2^3} = 2^{-3}$ $\quad$ $c^? = \sqrt[7]{c^3} = c^{\frac{3}{7}}$ $\quad$ $(\sqrt{b})^? = b^4 = \left((\sqrt{b})^2\right)^4 = (\sqrt{b})^8$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_3 0.13$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_3 0.13 = \frac{\ln 0.13}{\ln 3} = -1.857089$

3. (5pts) If $\log_a 5 = u$ and $\log_a 9 = v$, express in terms of u and v:

$\log_a \frac{9}{5} = \log_a 9 - \log_a 5$

$= v - u$

$\log_a 15 = \log_a (3 \cdot 5) = \log_a (\sqrt{9} \cdot 5)$

$= \log_a \sqrt{9} + \log_a 5 = \frac{1}{2}\log_a 9 + \log_a 5$

$= \frac{1}{2}v + u$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \frac{x^4}{9\sqrt{y^7}} = \log_3 x^4 - \log_3 9 - \log_3 y^{\frac{7}{2}}$

$= 4\log_3 x - 2 - \frac{7}{2}\log_3 y$

5. (6pts) Write as a single logarithm. Simplify if possible.

$3\log_2(x^{-2}y^4) - 4\log_2(x^2y^5) = \log_2 (x^{-2}y^4)^3 - \log_2 (x^2y^5)^4 = \log_2 (x^{-6}y^{12}) - \log_2 (x^8 y^{20})$

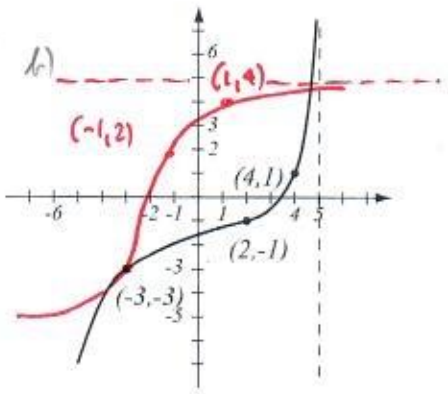
$= \log_2 \frac{x^{-6}y^{12}}{x^8 y^{20}} = \log_2 (x^{-14}y^{-8}) = \log_2 \frac{1}{x^{14}y^8}$

6. (4pts) Simplify.

$\log 10^{x-3} = x - 3$ $\quad$ $4^{\log_4(7x)} = 7x$

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.

a) Yes - it passes the horizontal line test



8. (9pts) Let $f(x) = \dfrac{4x-2}{2x+3}$.
a) Find the formula for f^{-1}.
b) Find the range of f^{-1}.

$y = \dfrac{4x-2}{2x+3}$

$y(2x+3) = 4x-2$

$2xy + 3y = 4x - 2$

$2xy - 4x = -3y - 2$

$x(2y-4) = -3y-2$

$x = \dfrac{-3y-2}{2y-4} = \dfrac{3y+2}{4-2y} = f^{-1}(y)$

Range f^{-1} = domain f

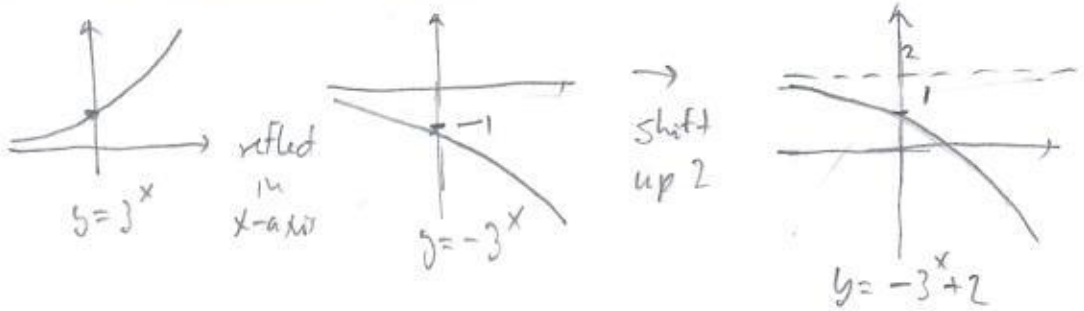
Can't have $2x+3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

Range f^{-1}: $(-\infty, -\frac{3}{2}) \cup (-\frac{3}{2}, \infty)$

9. (6pts) Using transformations, draw the graph of $f(x) = 2 - 3^x$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



Should be $\log_3(2x-7)\cdot\log_3(5-x)$

10. (6pts) Find the domain of the function $f(x) = \dfrac{\log_3(2x-7)}{\log_7(5-x)}$ and write it in interval notation.

Must have: $2x-7>0$ and $5-x>0$

$2x>7$ $\quad 5>x$

$x>\frac{7}{2}$ and $x<5$ $\quad (\frac{7}{2}, 5)$

11. (8pts) How much should you invest in an account bearing 3.1%, compounded quarterly, if you wish to have \$1,000 in five years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$1000 = P\left(1+\frac{0.031}{4}\right)^{4\cdot 5}$

$1000 = P\cdot 1.16696$

$P = \dfrac{1000}{1.16696} = 856.93$

Solve the equations.

12. (6pts) $16^{3x-2} = \left(\frac{1}{8}\right)^{x+1}$

$(2^4)^{3x-2} = (2^{-3})^{x+1}$

$2^{12x-8} = 2^{-3x-3}$

$12x-8 = -3x-3 \quad |+3x+8$

$15x = 5$

$x = \frac{5}{15} = \frac{1}{3}$

13. (4pts) $5^{2x} = 4$ $\quad |\ln$

$\ln 5^{2x} = \ln 4$

$2x\ln 5 = \ln 4$

$x = \dfrac{\ln 4}{2\ln 5} = 0.430677$

14. (10pts) $2^{2x} - 16 = 6\cdot 2^x$

$(2^x)^2 - 6\cdot 2^x - 16 = 0$ Let $u = 2^x$

$u^2 - 6u - 16 = 0$

$(u-8)(u+2) = 0$

$u = 8, -2$

$2^x = 8$

$x = 3$

$2^x = -2$

no sol.

$2^x > 0$

15. (12pts) The population of Fecund Grove was 14,000 in 2005 and 22,000 in 2011. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2005. Graph it on paper.
b) Find the predicted population in the year 2021.

a) 14000 in 2005

22000 in 2011

$P(t) = P_0 e^{kt} = 14e^{kt}$ (in thousands)

$22 = P(6) = 14e^{k\cdot 6}$

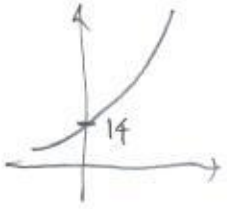
$22 = 14e^{6k}$

$\frac{22}{14} = e^{6k}$ | ln

$\ln\frac{22}{14} = 6k$

$k = \frac{\ln\frac{11}{7}}{6} = 0.0753309$

about 7.53%



Population in 2021 is

$P(16) = 14e^{0.0753305\cdot 16} = 46.728347$

About 46,728 people in 2021

Bonus (10pts) Let $f(x) = \dfrac{3}{1+e^{-x}}$.
a) Find the inverse function of f.
b) Show that $f^{-1}(f(x)) = x$.

a) $y = \frac{3}{1+e^{-x}}$

$\frac{y}{3} = \frac{1}{1+e^{-x}}$

$\frac{3}{y} = 1+e^{-x}$

$e^{-x} = \frac{3}{y} - 1$ | ln

$-x = \ln\left(\frac{3}{y}-1\right)$

$x = -\ln\left(\frac{3}{y}-1\right)$

$f^{-1}(y) = -\ln\left(\frac{3}{y}-1\right)$

b) $f^{-1}(f(x)) = f^{-1}\left(\frac{3}{1+e^{-x}}\right)$

$= -\ln\left(\frac{3}{\frac{3}{1+e^{-x}}} - 1\right) =$

$= -\ln\left(3\cdot\frac{1+e^{-x}}{3} - 1\right)$

$= -\ln\left(1+e^{-x}-1\right)$

$= -\ln e^{-x} = -(-x) = x$