College Algebra — Exam 3
MAT 140, Spring 2016 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $(1+i)^3 = (1+i)^2 \cdot (1+i) = (1+2i+i^2)(1+i) = 2i(1+i) = 2i + 2\underbrace{i^2}_{=-1}$

$= -2 + 2i$

2. (5pts) $\dfrac{2+5i}{4-i} = \dfrac{2+5i}{4-i} \cdot \dfrac{4+i}{4+i} = \dfrac{8+2i+20i+5\overbrace{i^2}^{=-1}}{4^2 - \underbrace{i^2}_{=-1}} = \dfrac{3+22i}{16+1} = \dfrac{3+22i}{17}$

3. (4pts) Simplify and justify your answer.

$i^{91} = i^{88} \cdot i^3 = (\underbrace{i^4}_{=1})^{22} \cdot i^3 = i^3 = \underbrace{i \cdot i}_{-1} \cdot i = -i$

4. (6pts) Solve the equation by completing the square.

$x^2 - 10x + 33 = 0 \quad | + 5^2$

$x^2 - 2 \cdot x \cdot 5 + 5^2 + 33 = 5^2$

$(x-5)^2 = 25 - 33$

$(x-5)^2 = -8$

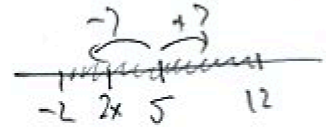
$x - 5 = \pm\sqrt{-8}$

$x = 5 \pm \sqrt{8}\, i = 5 \pm 2\sqrt{2}\, i$

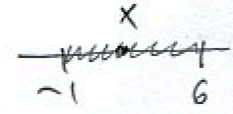
5. (6pts) Solve the inequality. Write the solution in interval form.

$|2x - 5| < 7$

distance from $2x$ to $5 < 7$



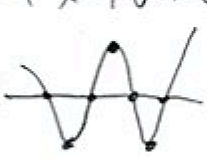
$\xrightarrow{\div 2}$



$(-1, 6)$

6. (6pts) Let $P(x)$ be a polynomial.
a) If the graph of P has 3 x-intercepts, what can you say about the degree of P? At least 3
b) If the graph of P has 5 turning points, what can you say about the degree of P? At least 6
c) Can the graph of P have 4 x-intercepts and 2 turning points? Explain why or why not.

No - between every two x-intercepts there is at least one turning point, so with 4 x-intercepts, there are at least 3 turning points.



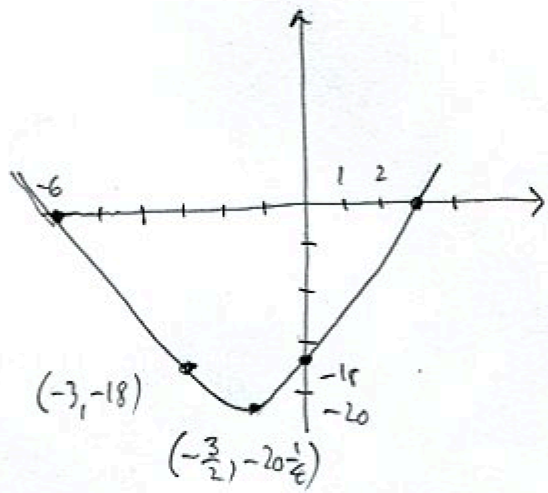
7. (12pts) The quadratic function $f(x) = x^2 + 3x - 18$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) y-int: $f(0) = -18$

x-int: $x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x = -6, 3$



b) vertex: $h = -\frac{b}{2a} = -\frac{3}{2 \cdot 1} = -\frac{3}{2}$

$k = \left(-\frac{3}{2}\right)^2 + 3\cdot\left(-\frac{3}{2}\right) - 18 = \frac{9}{4} - \frac{9}{2} - 18 = -\frac{9}{4} - 18 = \frac{-9-72}{4} = -\frac{81}{4} = -20\frac{1}{4}$

Solve the equations:

8. (8pts) $1 - \dfrac{6}{x+1} = \dfrac{4x+16}{x^2+5x+4}$ $\Big| \cdot (x+1)(x+4)$

$(x+1)(x+4)$

$(x+1)(x+4) - 6(x+4) = 4x+16$

$x^2 + 5x + 4 - 6x - 24 = 4x + 16$ $\big| -4x - 16$

$x^2 - 5x - 36 = 0$

$(x-9)(x+4) = 0$

$x = 9, \cancel{-4}$

not a solution,
gives 0 in denominator

9. (8pts) $x = \sqrt{x+9} + 3$

$x - 3 = \sqrt{x+9}$ $\big|^2$

$x^2 - 2 \cdot x \cdot 3 + 3^2 = x + 9$

$x^2 - 6x + 9 = x + 9$ $\big| -x - 9$

$x^2 - 7x = 0$

$x(x-7) = 0$

$x = 0, 7$

Check: $x=0$: $0 \stackrel{?}{=} \sqrt{9} + 3$ no

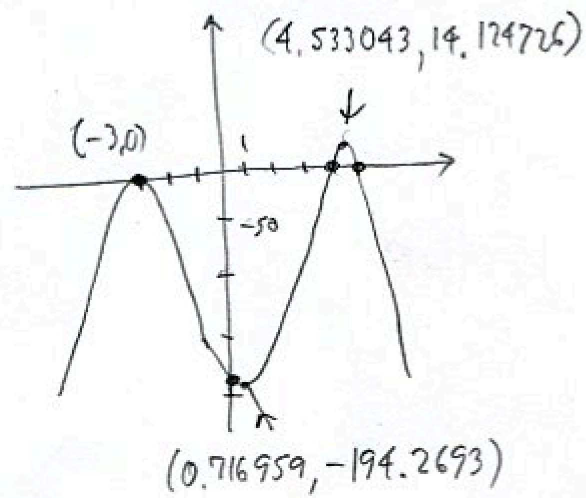
$x = 7$: $7 \stackrel{?}{=} \sqrt{16} + 3$ yes

$\boxed{x=7}$ is the only solution

10. (14pts) The polynomial $f(x) = -(x+3)^2(x-4)(x-5)$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

a) like $-(x)^2(x)\cdot(x) = -x^4$ $\cap$

c)



b)

zero	−3	4	5
mult.	2	1	1

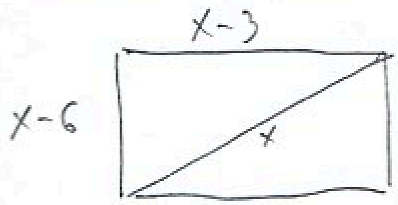
y-int: $f(0) = -9\cdot(-4)(-5) = -180$

d) Local max: $f(-3) = 0$

$f(4.533043) = 14.124726$

Local min: $f(0.716959) = -194.2693$

11. (12pts) In a rectangle, the sides are 3 ft and 6ft shorter than the diagonal. What are the dimensions of the rectangle?



x = length of a diagonal

Pythagorean Theorem

$(x-6)^2 + (x-3)^2 = x^2$

$x^2 - 2\cdot x\cdot 6 + 6^2 + x^2 - 2\cdot x\cdot 3 + 9 = x^2$

$2x^2 - 18x + 45 = x^2 \quad | -x^2$

$x^2 - 18x + 45 = 0$

$(x-15)(x-3) = 0$

$x = 15$ or $x = 3$

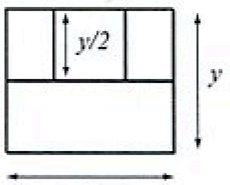
not a solution, since $x-6$ is negative

rectangle is 9×12

12. (14pts) A company is building a conference center whose floorplan is below. It has budgeted enough money to build 750 feet of walls, and its goal is to maximize the total area of the building.

a) Express the total area of the building as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the biggest possible total area, and what is the biggest possible total area?



a) $3x+2y+2\cdot\frac{y}{2}=750$

$3x+3y=750$

$x+y=250$

$y=250-x$

$A=x\cdot y=x(250-x)=-x^2+250x$

Domain must have: $x\geq 0$

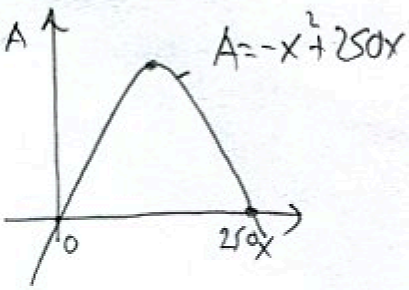
$y\geq 0$

$250-x\geq 0$

$x\leq 250$

Domain: $[0,250]$

b)



$h=-\frac{b}{2a}=-\frac{250}{2\cdot(-1)}=125$

$k=-125^2+250\cdot 125=15{,}625$

Dimensions: 125×125 ft

Max area: 15,625 sq. ft

Bonus. (10pts) Solve the following equation for a complex number z. Your answer should be in form $a+bi$. *Hint: work like you would with real numbers to isolate* z.

$4+iz=2z-3i \quad \vert -2z-4$

$iz-2z=-4-3i$

$z(i-2)=-4-3i$

$z=\frac{-4-3i}{i-2}=\frac{4+3i}{2-i}\cdot\frac{2+i}{2+i}=\frac{8+6i+4i+\underbrace{3i^2}_{-3}}{2^2-i^2}=\frac{5+10i}{4-(-1)}=\frac{5+10i}{5}=1+2i$