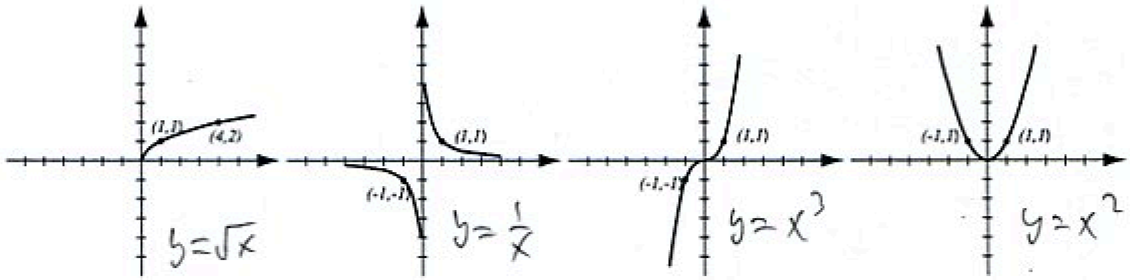
College Algebra — Exam 2
MAT 140, Spring 2016 — D. Ivanšić

Name: Saul Ocean

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=\sqrt{x}$ $\quad$ $y=\frac{1}{x}$ $\quad$ $y=x^3$ $\quad$ $y=x^2$

2. (20pts) Let $f(x)=\dfrac{x^2}{x-7}$, $g(x)=\sqrt{2x+9}$.

Find the following (simplify where possible):

$(f-g)(8) = f(8)-g(8) = \frac{8^2}{8-7} - \sqrt{2\cdot 8+9}$

$= \frac{64}{1} - \sqrt{25} = 59$

$(fg)(1) = f(1)\cdot g(1) = \frac{1^2}{1-7}\cdot\sqrt{2\cdot 1+9}$

$= -\frac{1}{6}\cdot\sqrt{11} = -\frac{\sqrt{11}}{6}$

$\frac{g}{f}(x) = \frac{g(x)}{f(x)} = \frac{\sqrt{2x+9}}{\frac{x^2}{x-7}}$

$= \sqrt{2x+9}\cdot\frac{x-7}{x^2} = \frac{(x-7)\sqrt{2x+9}}{x^2}$

$(g\circ f)(9) = g(f(9)) = g\left(\frac{9^2}{9-7}\right) = g\left(\frac{81}{2}\right)$

$= \sqrt{2\cdot\frac{81}{2}+9} = \sqrt{90}$

$(f\circ g)(x) = f(g(x)) = f(\sqrt{2x+9}) =$

$= \frac{\sqrt{2x+9}^2}{\sqrt{2x+9}-7} = \frac{2x+9}{\sqrt{2x+9}-7}$

The domain of $f+g$ in interval notation

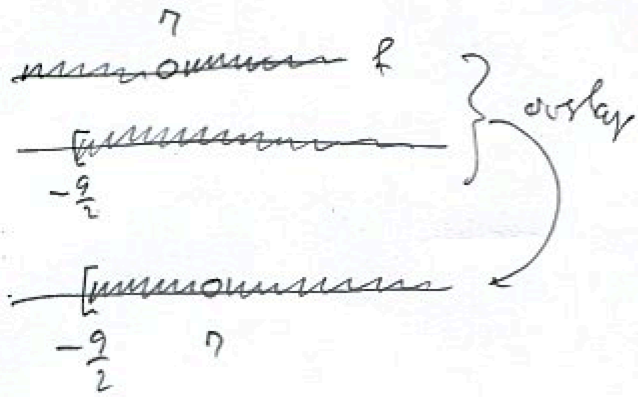
Domain of f: $\quad$ Domain of g:

Can't have $\quad$ Must have

$x-7=0$ $\quad$ $2x+9\geq 0$

$x=7$ $\quad$ $2x\geq -9$

$x\geq -\frac{9}{2}$



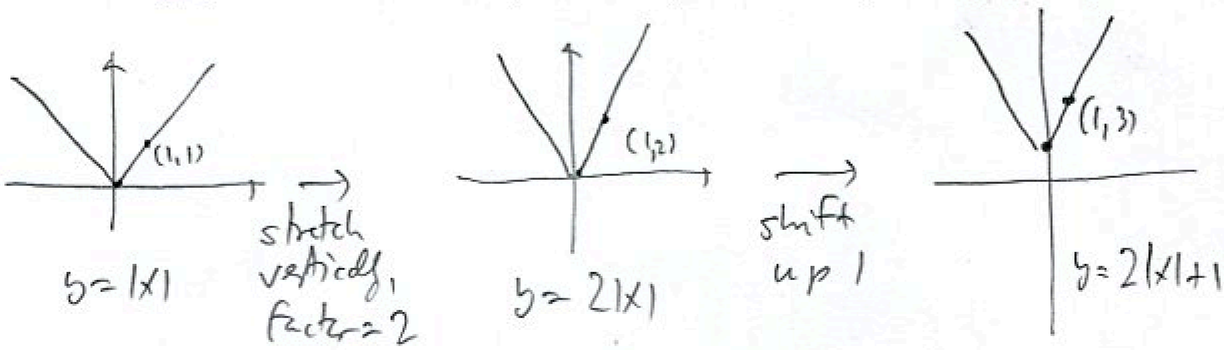
$[-\frac{9}{2}, 7)\cup(7,\infty)$

3. (6pts) Consider the function $h(x) = \sqrt[3]{5x+2}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

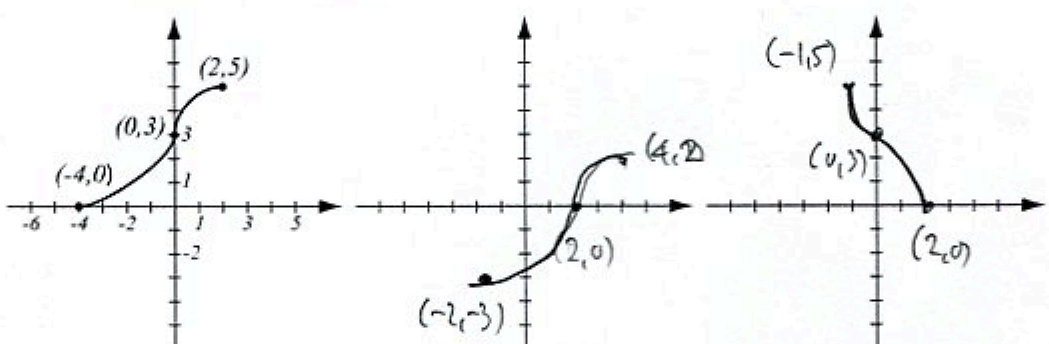
$g(x) = 5x+2$ $\qquad$ $g(x) = 5x$

$f(x) = \sqrt[3]{x}$ $\qquad$ $f(x) = \sqrt[3]{x+2}$

4. (8pts) Using transformations, draw the graph of $f(x) = 2|x| + 1$. Explain how you transform the graph of a basic function (which one?) in order to get the graph of f.



5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(x-2)-3$ and $f(-2x)$ and label all the relevant points.



shift right 2 down 3

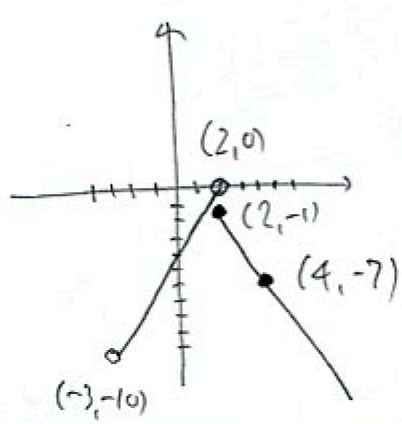
stretch horizontally, factor = $\frac{1}{2}$ reflect in y-axis

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x-4, & \text{if } -3 < x < 2 \\ -3x+5, & \text{if } x \geq 2 \end{cases}$$

x	2x−4
−3	−10
2	0

x	−3x+5
2	−1
4	−7



7. (8pts) Simplify.

$$\frac{2x}{x^2+2x-3} - \frac{x-3}{x^2+4x-5} = \frac{2x(x+5)-(x-3)(x+3)}{(x-1)(x+3)(x+5)} = \frac{2x^2+10x-(x^2-9)}{(x-1)(x+3)(x+5)}$$

$(x-1)(x+3)$ $\quad$ $(x-1)(x+5)$

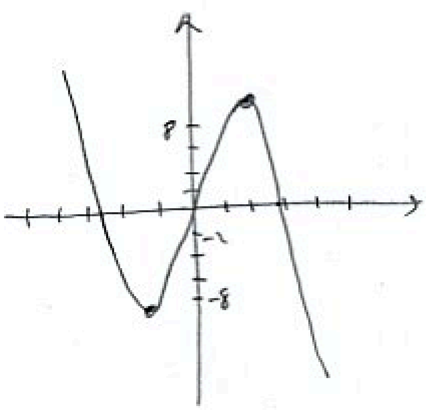
x^2-3^2

$$= \frac{x^2+10x+9}{(x-1)(x+3)(x+5)} = \frac{(x+1)(x+9)}{(x-1)(x+3)(x+5)}$$

nothing reduces

8. (18pts) Let $f(x) = -x^3 + 8x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether the function is odd, even, or neither
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function.
e) State the intervals where the function is increasing and where it is decreasing.

a)



d) $f(-1.632992) = -8.709297$ is local min

$f(1.632992) = 8.709297$ is local max

e) Increasing on

$(-1.632992, 1.632992)$

Decreasing on

$(-\infty, -1.632992)$ and $(1.632992, \infty)$

b) $f(-x) = -(-x)^3 + 8(-x)$

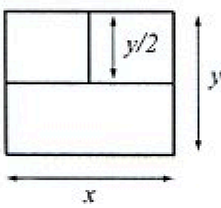
$= -(-x^3) - 8x$

$= x^3 - 8x = -f(x)$

odd function

c) Graph is symmetric wrt. origin.

9. (14pts) Georgina is planning a simple 3-room house with an area of 900 square feet (see picture). She wishes to minimize the total length of the walls.
a) Express the total length of the walls as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the length function here and find its minimum. What are the dimensions of the house for which the total length of the walls is minimal? What is the minimal wall length?

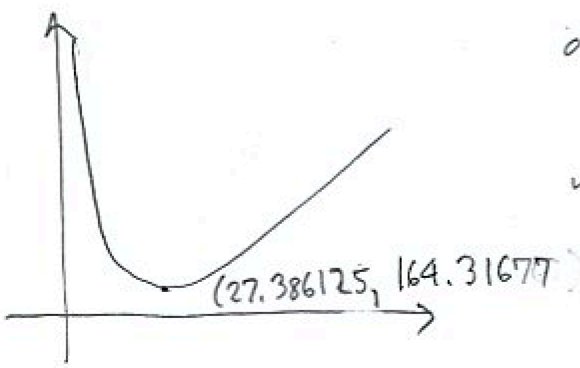


$\ell = 3x + 2y + \frac{y}{2} = 3x + \frac{5}{2}y = 3x + \frac{5}{2}\cdot\frac{900}{x} = 3x + \frac{2250}{x}$

$xy = 900$

$y = \frac{900}{x}$

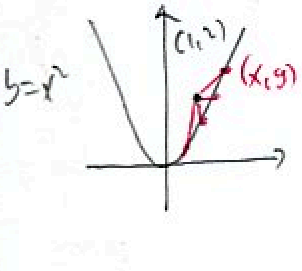
Domain: $x > 0$, so $(0, \infty)$



dimensions: 27.386125 × 32.863356

minimal wall length = 164.31677

Bonus. (10pts) Recall that the distance between points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Use this to find the point on the curve $y = x^2$ that is closest to the point $(1, 2)$. *Hint: minimize the distance from a point (x, y) on the curve to the point $(1, 2)$. Make it a function only of x.*



distance from (x,y) to $(1,2)$ is

$d = \sqrt{(x-1)^2 + (y-2)^2} = \sqrt{(x-1)^2 + (x^2-2)^2} = \sqrt{x^2 - 2x + 1 + x^4 - 4x^2 + 4}$

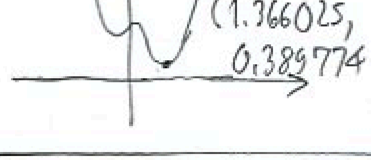
$y = x^2$ since (x,y) is on curve $y = x^2$

$d(x) = \sqrt{x^4 - 3x^2 - 2x + 5}$

$x \qquad x^2$

Smallest distance is from point $(1.366025, 1.866024)$

Graph,



and the distance is 0.389774