Calculus 1 — Exam 1
MAT 250, Spring 2015 — D. Ivanšić

Name: _____

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -3} f(x) =$

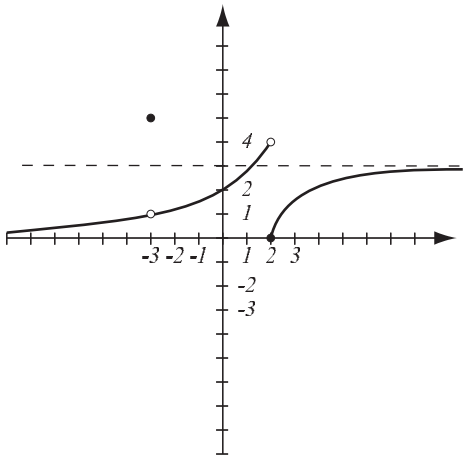
$\lim_{x\to 2^+} f(x) =$

$\lim_{x\to 2^-} f(x) =$

$\lim_{x\to 2} f(x) =$

$\lim_{x\to \infty} f(x) =$

$\lim_{x\to -\infty} f(x) =$



List points where f is not continuous and justify why it is not continuous at those points.

2. (4pts) Briefly explain why the function $f(x) = \dfrac{x+3}{3x-2}$ is continuous on its domain.

3. (10pts) Find $\lim_{x\to 0} x^4 \left((7+\cos\left(\frac{1}{x^3}\right)\right)$. Use the theorem that rhymes with what a doctor may cure.

Find the following limits algebraically. Do not use the calculator.

4. (5pts) $\lim_{x \to 7} \dfrac{x-7}{x^2-2x-35} =$

5. (7pts) $\lim_{x \to 16} \dfrac{\sqrt{x}-4}{x-16} =$

6. (7pts) $\lim_{x \to 0} \dfrac{\sin(3x)}{5x} =$

7. (7pts) $\lim_{x \to \infty} \dfrac{4x^3-3x^2-7x+2}{7x^3-x^2+5x} =$

8. (6pts) $\lim_{x \to 5^+} \dfrac{4-2x}{x-5} =$

9. (10pts) Use the Intermediate Value Theorem to show that the equation $x^3+2x = 4\sqrt{x}+2$ has at least one solution.

10. (10pts) Consider the limit $\lim_{x\to 0} \dfrac{5^x - 1}{x}$. Use your calculator to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.

x	$\dfrac{5^x - 1}{x}$

x	$\dfrac{5^x - 1}{x}$

11. (4pts) Consider the limit below, representing a derivative $f'(a)$: find f and a.

$$\lim_{h\to 0} \frac{(3+h)^4 - 81}{h}$$

12. (14pts) The amount of water (in gallons) in a 100-gallon tank that is draining at the bottom is given by $V(t) = t^2 - 20t + 100$, where t is in minutes, $0 \leq t \leq 10$.
a) What is the average rate of draining from $t = 2$ to $t = 5$? What are the units?
b) What is the instantaneous rate of draining when $t = 2$? What are the units?

Bonus. (10pts) Consider the limit $\lim_{x \to \sqrt{3}} \dfrac{x^2 - 3}{x - \sqrt{3}}$.

a) Use your calculator to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.
b) Find the limit algebraically.

x	$\dfrac{x^2 - 3}{x - \sqrt{3}}$

Calculus 1 — Exam 2
MAT 250, Spring 2015 — D. Ivanšić

Name: _____

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\dfrac{d}{dx}\left(3x^7 - \dfrac{5}{x^3} - \sqrt[5]{x^3} - 7c^2\right) =$

2. (5pts) $\dfrac{d}{dt}\,(t^2+3)\cos t =$

3. (6pts) $\dfrac{d}{dx}\,\dfrac{2x-7}{x^2+4x-5} =$

4. (6pts) $\dfrac{d}{d\theta}\,(\sec^2\theta - \tan^2\theta) =$

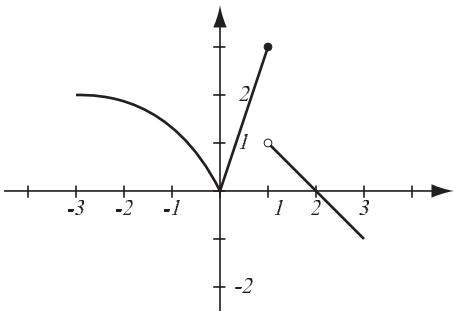
5. (7pts) $\dfrac{d}{dx}\left(ax + \sqrt{bx^3 - 7x}\right)^5 =$

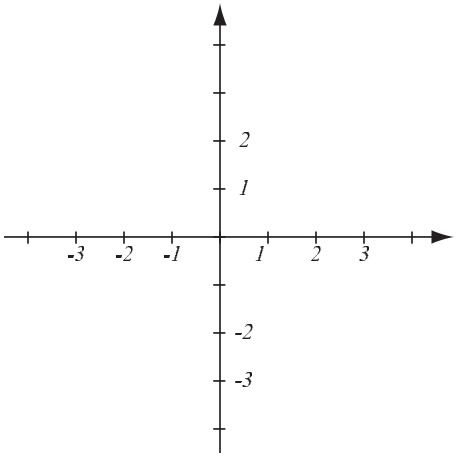
6. (8pts) Let $g(x) = xf(x)$ and $h(x) = f(x^2)$.
a) Find the general expressions for $g'(x)$ and $h'(x)$.
b) Use the table of values below to find $g'(3)$ and $h'(2)$.

x	1	2	3	4
$f(x)$	7	3	0	1
$f'(x)$	-2	1	-2	3

7. (10pts) The graph of the function $f(x)$ is shown at right.

a) Where is $f(x)$ not differentiable?
b) Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.





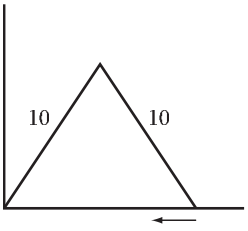
8. (15pts) Let $f(x) = 3x^2 + 5x - 1$.
a) Use the limit definition of the derivative to find the derivative of the function.
b) Check your answer by taking the derivative of f using differentiation rules.
c) Write the equation of the tangent line to the curve $y = f(x)$ at point $(1, 7)$.

9. (9pts) A snowball is thrown upwards from ground level with initial velocity 20m/s. Its position is given by the formula $s(t) = -5t^2 + 20t$.
a) Write the formula for the velocity of the snowball at time t.
b) When does the snowball reach its maximum height and what is it?

10. (12pts) Use implicit differentiation to find y'.

$x^2 + y^2 = \sin x \cos y$

11. (16pts) A folding ladder whose sides are 10ft long has one end against a wall. If the other end is pushed toward the wall at rate 1/4 foot per second, how fast is the top of the ladder rising when the pushed end is 6 feet away from the wall?



Bonus. (10pts) The Energizer Bunny moves along a straight road so that his position function is $s(t) = t^3 - 15t^2 + 48t + 2$.
a) Find the velocity and acceleration functions and sketch their graphs.
b) When is the Bunny moving forward? Backward?
c) Use the information you found above to sketch the Bunny's path.
d) What is his velocity when acceleration is 0?

Calculus 1 — Exam 3
MAT 250, Spring 2015 — D. Ivanšić

Name: _____

Show all your work!

Differentiate and simplify where appropriate:

1. (3pts) $\dfrac{d}{dx}\, e^{x^2+3x-1} =$

2. (4pts) $\dfrac{d}{dx}\, \ln(\tan^2 x) =$

3. (6pts) $\dfrac{d}{dt}\, \dfrac{t^2-3t}{7^t} =$

4. (7pts) $\dfrac{d}{dx}\, \ln \dfrac{\sin x + \cos x}{\sin x - \cos x} =$

5. (8pts) $\dfrac{d}{du}\, \left(u \arctan u - \frac{1}{2} \ln(1+u^2)\right) =$

6. (10pts) Use logarithmic differentiation to find the derivative of $y = x^{\sqrt{x}}$.

7. (4pts) Draw the graphs of e^x, $\ln x$ and $\arctan x$ (each in its coordinate system).

Find the limits algebraically. Graphs of basic functions will help, as will L'Hospital's rule, where appropriate.

8. (2pts) $\lim_{x \to -\infty} 5^x =$

9. (6pts) $\lim_{x \to 0^+} \arctan\left(4 - \frac{1}{x}\right) =$

10. (6pts) $\lim_{x \to 0} \frac{1 - \cos x}{x^2} =$

11. (6pts) $\lim_{x \to 0^+} x^3 \ln x =$

12. (10pts) $\lim_{x \to \infty} (x^2 + 3x - 1)^{\frac{1}{x}} =$

13. (10pts) Let $f(x) = \sqrt[3]{x}$.
a) Write the linearization of $f(x)$ at $a = 8$.
b) Use the linearization to estimate $\sqrt[3]{8.3}$ and compare to the calculator value of 2.024694.

14. (10pts) Radius of a sphere r is measured to be 10 meters, with maximum error 5 centimeters. Use differentials to estimate the maximum possible error, the relative error and the percentage error when computing the surface area A of the sphere ($A = 4\pi r^2$, leave your answer in terms of π).

15. (8pts) Let $f(x) = e^x + 3x + 4$. Use the theorem on derivatives of inverses to find $(f^{-1})'(5)$.

Bonus. (10pts) Find the limit. (Note: for small $x > 0$, $\ln x < 0$, so we need a minus to ensure that the base is a positive number).

$\lim_{x \to 0+} (-\ln x)^{\ln(x+1)} =$

Calculus 1 — Exam 4
MAT 250, Spring 2015 — D. Ivanšić

Name: _____

Show all your work!

1. (30pts) Let $f(x) = \dfrac{x^2}{x^2+1}$. Draw an accurate graph of f by following the guidelines.

a) Find the intervals of increase and decrease, and local extremes.

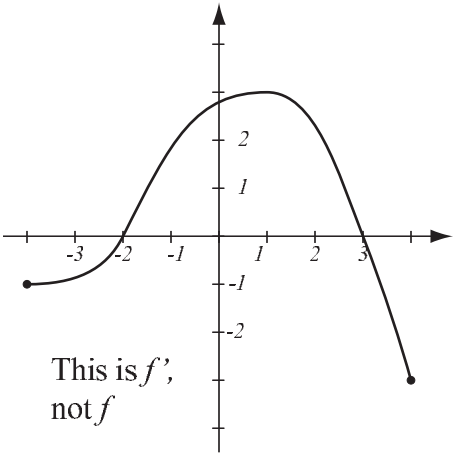
b) Find the intervals of concavity and points of inflection.

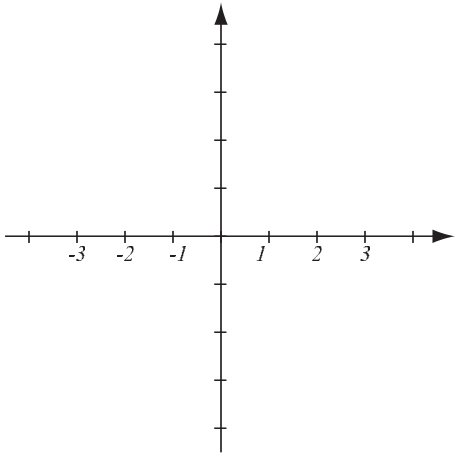
c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

d) Use information from a)–d) to sketch the graph.

2. (14pts) Let $f(x) = 24x^{\frac{1}{2}} - 2x^{\frac{3}{2}}$. Find the absolute minimum and maximum values of f on the interval $[1, 9]$.

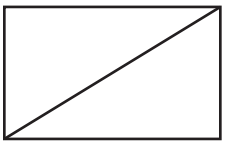
3. (16pts) Let f be continuous on $[-4, 4]$. The graph of its derivative f' is drawn below. Use the graph to answer (sign charts may help):
a) What are the intervals of increase and decrease of f? Where does f have a local minimum or maximum?
b) What are the intervals of concavity of f? Where does f have inflection points?
c) Use the information gathered in a) and b) to sketch the graph of f at right, if $f(-4) = 0$.





4. (16pts) Let $f(x) = \sin^2 x$, $0 \leq x \leq 2\pi$. Find the intervals of concavity and points of inflection for f.

5. (24pts) Among all rectangles of area 100 square meters, find the one which has the shortest diagonal.



Bonus. (10pts) Suppose $f(x) > 0$ and f is concave up. Let $g(x) = (f(x))^2$.
a) Find the expression for $g''(x)$.
b) Show that g is concave up.

Calculus 1 — Exam 5
MAT 250, Spring 2015 — D. Ivanšić

Name: _____

Show all your work!

Find the following antiderivatives.

1. (3pts) $\displaystyle\int \frac{1}{\sqrt[3]{x^2}}\,dx =$

2. (3pts) $\displaystyle\int \frac{5}{\sqrt{1-x^2}}\,dx =$

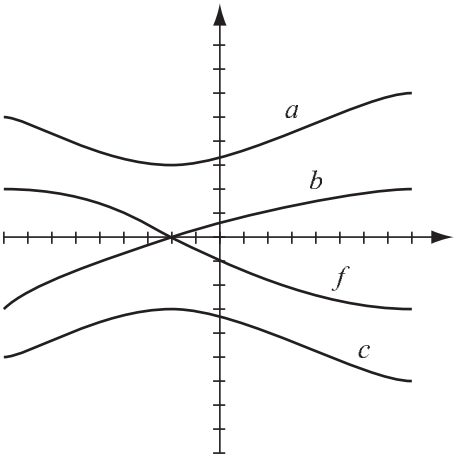
3. (3pts) $\displaystyle\int e^{3x+7}\,dx =$

4. (7pts) $\displaystyle\int \frac{u^2-u+1}{\sqrt{u}}\,du =$

5. (7pts) Find $f(x)$ if $f'(x) = \cos(3x) + \sec^2 x$ and $f(0) = 4$.

6. (8pts) Find $f(x)$ if $f''(x) = \dfrac{4}{x^3}$, $f'(1) = 3$ and $f(2) = -2$.

7. (6pts) The graph of a function f is shown. Which of the other graphs is an antiderivative of f and why?



8. (15pts) Find $\int_0^4 x - 1\, dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture and use area of triangles.
b) Using the Evaluation Theorem.

Use the substitution rule in the following integrals:

9. (8pts) $\int (3x^2 - 2x)\sqrt{x^3 - x^2 + 1}\, dx =$

10. (10pts) $\int_0^{\frac{\pi}{2}} \frac{\sin x}{2 + \cos x}\, dx =$

11. (10pts) $\int_3^5 \frac{e^{\frac{1}{x}}}{x^2}\, dx =$

12. (10pts) Evaluate the following integral by breaking it up into two integrals without absolute value and evaluating each one. The graph of $y = |x - 2|$ might help.

$\int_1^5 |x - 2|\, dx =$

13. (10pts) The rate at which water is flowing into a tank is $-t^2+10t-9$ liters per minute.
a) Use the Net Change Theorem to find by how much the volume of water in the tank has changed from $t=0$ to $t=6$.
b) If at time $t=0$ there were 23 liters of water in the tank, how many were there at time $t=6$?

Bonus. (10pts) A rocket takes off vertically from the ground, accelerating at constant acceleration. If at time $t=10$ seconds it is at height 900 meters, what was its acceleration?