**Calculus 1 — Exam 4**
**MAT 250, Spring 2013 — D. Ivanšić**

**Name:** _____

*Show all your work!*

Differentiate and simplify where appropriate:

**1.** (3pts) $\dfrac{d}{dx}\, e^{\cos x} =$

**2.** (4pts) $\dfrac{d}{dx}\, \ln(\sec x + 1) =$

**3.** (6pts) $\dfrac{d}{du}\, \dfrac{u+1}{3^u} =$

**4.** (7pts) $\dfrac{d}{dt}\, \ln \sqrt[6]{\dfrac{t^2 - 4t + 1}{3t^{-}1}} =$

**5.** (8pts) $\dfrac{d}{dx}\, \left( \dfrac{1}{2} \arcsin x + \dfrac{1}{2} x \sqrt{1 - x^2} \right) =$

**6.** (10pts) Use logarithmic differentiation to find the derivative of $y = (x^2 - 4x + 7)^{\sin x}$.

Find the limits. Use L'Hospital's rule where appropriate.

**7.** (6pts) $\lim_{x\to 0^+} \arctan\left(\frac{x+2}{x}\right) =$

**8.** (8pts) $\lim_{x\to 0} \frac{\sin x - x}{x^3} =$

**9.** (6pts) $\lim_{x\to \infty} xe^{-x} =$

**10.** (10pts) $\lim_{x\to \infty} (1+x^2)^{\frac{1}{x}} =$

**11.** (10pts) Find the critical points of the function $f(x) = x^{\frac{1}{3}}(x^2 + 9x)$.

**12.** (12pts) Let $f(x) = (x^2 - 3x - 3)e^x$. Find the absolute minimum and maximum values of $f$ on the interval $[0, 4]$.

**13.** (10pts) Suppose $\theta$ is given implicitly as a function of $x$ by $\tan\theta = \sqrt{x^2-1}$.
a) Use implicit differentiation to find $\theta'$.
b) Using a trigonometric picture, express $\theta'$ only in terms of $x$.

**Bonus.** (10pts) Suppose that $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = \infty$. Recall that we say $f(x)$ grows to infinity slower than $g(x)$ if $\lim_{x\to\infty} \dfrac{f(x)}{g(x)} = 0$.

a) If $\lim_{x\to\infty} f(x) = \infty$ , show that $\lim_{x\to\infty} \dfrac{\ln f(x)}{f(x)} = 0$.

b) If $\lim_{x\to\infty} f(x) = \infty$, show that there exists a function that grows to infinity slower than $f(x)$. This means there does not exist a "slowest-growing" function to infinity.