

Calculus 1 — Exam 2
MAT 250, Spring 2013 — D. Ivanšić

Name: _____
Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\frac{d}{dx} \left(4x^4 + \frac{4}{x^2} - x^2\sqrt{x} + \sqrt[4]{\pi} \right) =$

2. (5pts) $\frac{d}{du} (u^2 - 3u)\sqrt{4u - 7} =$

3. (6pts) $\frac{d}{dx} \frac{x - 4}{x^2 - 3x + 1} =$

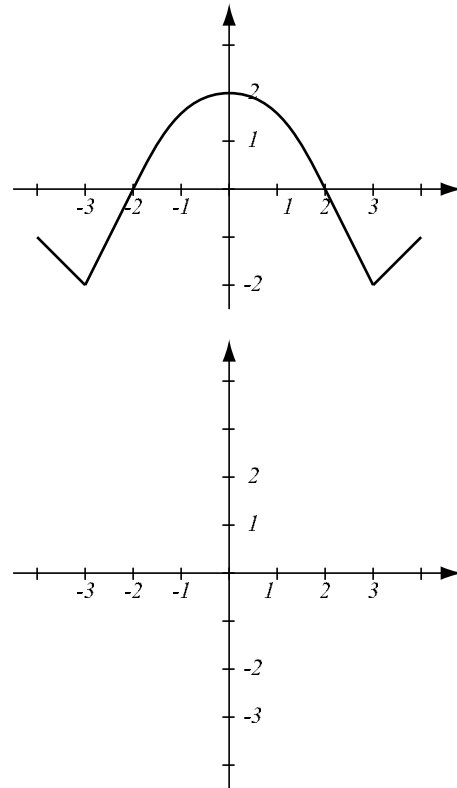
4. (6pts) $\frac{d}{d\theta} (\cos^2 \theta - \sin^2 \theta) =$

5. (7pts) $\frac{d}{dx} \sqrt{\tan(x^4 + x^2 + 1)} =$

6. (8pts) Let $h(x) = f(x) \sin x$. Find the general expressions for $h'(x)$ and $h''(x)$ and simplify where appropriate.

7. (12pts) The graph of the function $f(x)$ is shown at right.

- Where is $f(x)$ not differentiable?
- Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.
- Is $f(x)$ odd or even? How about $f'(x)$?



8. (16pts) Let $f(x) = \frac{1}{3x+2}$.

- Use the limit definition of the derivative to find the derivative of the function.
- Check your answer by taking the derivative of f using rules.
- Write the equation of the tangent line to the curve $y = f(x)$ at point $(3, \frac{1}{11})$.

9. (16pts) A first-generation iPhone is thrown upwards from ground level with initial velocity 40m/s (what else to do with a first-generation device!). Its position is given by the formula $s(t) = -5t^2 + 40t$.

- a) Write the formula for the velocity of the iPhone at time t .
- b) When does the iPhone reach height 60 meters?
- c) What is the velocity of the iPhone when it reaches height 60 meters on its way up? On its way down?
- d) What is the height of the iPhone when its velocity is 10m/s?

10. (6pts) Consider the limit below. It represents a derivative $f'(a)$.

- a) Find f and a .
- b) Once you've found f and a , find the limit — it is equal to $f'(a)$!

$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$$

11. (12pts) The temperature (in degrees Celsius) of water being heated in a pan is given by $f(t) = 10 + 30\sqrt{t}$, where t is in minutes.

a) What is the average rate of warming from $t = 4$ to $t = 6$? What are the units?

b) What is the instantaneous rate of warming when $t = 4$? What are the units?

c) Draw the graph of f and state the geometric interpretation of the numbers you got above.

Bonus. (10pts) Take the derivative of the function below and simplify. Go to town!

$$\frac{d}{dx} \sqrt{\frac{x + \sqrt{x}}{x - \sqrt{x}}} =$$