Calculus 1 — Exam 1
MAT 250, Spring 2013 — D. Ivanšić

Name: ______________________________

Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim\limits_{x\to -2^-} f(x) =$

$\lim\limits_{x\to -2^+} f(x) =$

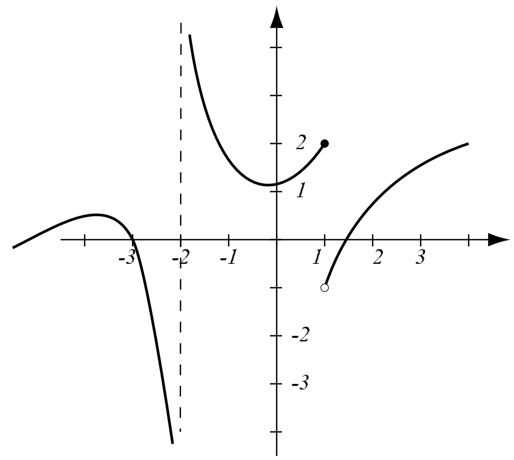
$\lim\limits_{x\to -2} f(x) =$

$\lim\limits_{x\to 1^+} f(x) =$

$\lim\limits_{x\to 1} f(x) =$

$f(1) =$

List points where f is not continuous and justify why it is not continuous at those points.



2. (8pts) Let $\lim\limits_{x\to 2} f(x) = 3$ and $\lim\limits_{x\to 2} g(x) = -1$. Use limit laws to find the limit below and show each step.

$$\lim_{x\to 2} \sqrt{\frac{xf(x)-4}{x^3+g(x)}} =$$

3. (10pts) Find $\lim\limits_{x\to 0} \dfrac{x^2}{4+\sin\left(\frac{1}{x}+3\right)}$. Use the theorem that rhymes with an exclamation conveying surprise and derision.

Find the following limits algebraically. Do not use the calculator.

4. (5pts) $\lim_{x \to 4} \dfrac{x^2 - 16}{x^2 - 5x + 4} =$

5. (7pts) $\lim_{x \to 13} \dfrac{\sqrt{x+3} - 4}{x - 13} =$

6. (6pts) $\lim_{x \to 0} \dfrac{\tan x}{x} =$

7. (7pts) $\lim_{x \to \infty} \dfrac{5x^2 - 3x + 1}{4x^3 - 4x^2 + 7} =$

8. (5pts) $\lim_{x \to 3^+} \dfrac{2x + 1}{3 - x} =$

9. (14pts) Use your calculator to find an interval of length at most 0.01 that contains the solution of the equation $x^3 - 4x^2 + 3x = 8$. Use the Intermediate Value Theorem to justify why your interval contains the solution.

10. (10pts) Consider the limit $\lim_{x \to 1} \dfrac{2^x - 2}{x - 1}$. Use your calculator to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.

x	$\dfrac{2^x - 2}{x - 1}$

11. (12pts) Draw the graph of a function, defined on the interval $(-3, 4)$ that exhibits the following features:

$\lim_{x\to\infty} f(x) = 3$
$\lim_{x\to-\infty} f(x) = 1$
$\lim_{x\to 0^-} f(x) = 4$
$\lim_{x\to 0^+} f(x) = -1$
$f(x)$ is left-continuous at $x = 0$

Bonus. (10pts) Show that $\frac{0}{0}$ is an indeterminate form. That is, come up with three pairs of functions $f(x)$, $g(x)$ such that $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ in every case, yet $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ is different for the three cases. (Think of simple functions.)