Calculus 1 — Exam 2
MAT 250, Spring 2015 — D. Ivanšić

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Show all your work!

Differentiate and simplify where appropriate:

constant

1. (6pts) $\frac{d}{dx}\left(3x^7 - \frac{5}{x^3} - \sqrt[5]{x^3} - 7c^2\right) = \frac{d}{dx}\left(3x^7 - 5x^{-3} - x^{\frac{3}{5}} - 7c^2\right)$

$= 21x^6 + 15x^{-4} - \frac{3}{5}x^{-\frac{2}{5}} = 21x^6 + \frac{15}{x^4} - \frac{3}{5\sqrt[5]{x^2}}$

2. (5pts) $\frac{d}{dt}(t^2+3)\cos t = 2t\cos t + (t^2+3)(-\sin t)$

$= 2t\cos t - (t^2+3)\sin t$

3. (6pts) $\frac{d}{dx}\frac{2x-7}{x^2+4x-5} = \frac{2(x^2+4x-5)-(2x-7)(2x+4)}{(x^2+4x-5)^2}$

$= \frac{2x^2+8x-10-(4x^2-6x-28)}{(x^2+4x-5)^2} = \frac{-2x^2+14x+18}{(x^2+4x-5)^2} = \frac{-2(x^2-7x-9)}{(x^2+4x-5)^2}$

4. (6pts) $\frac{d}{d\theta}(\sec^2\theta - \tan^2\theta) = 2\sec\theta\sec\theta\tan\theta - 2\tan\theta\sec^2\theta = 0$

5. (7pts) $\frac{d}{dx}\left(ax+\sqrt{bx^3-7x}\right)^5 = 5\left(ax+\sqrt{bx^3-7x}\right)^4\left(a+\frac{1}{2\sqrt{bx^3-7x}}(3bx^2-7)\right)$

$= 5\left(ax+\sqrt{bx^3-7x}\right)^4\left(a+\frac{3bx^2-7}{2\sqrt{bx^3-7x}}\right)$

6. (8pts) Let $g(x) = xf(x)$ and $h(x) = f(x^2)$.
a) Find the general expressions for $g'(x)$ and $h'(x)$.
b) Use the table of values below to find $g'(3)$ and $h'(2)$.

x	1	2	3	4
$f(x)$	7	3	0	1
$f'(x)$	-2	1	-2	3

when $x=3$ b)

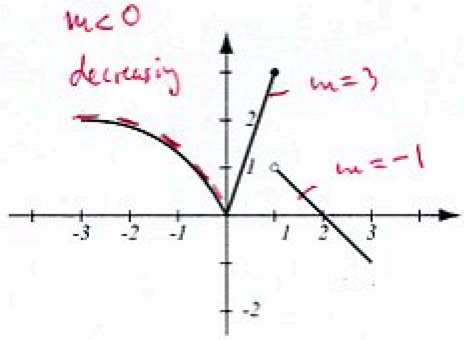
a) $(xf(x))' = f(x) + xf'(x) \longrightarrow f(3)+3\cdot f'(3) = 0+3\cdot(-2) = -6$

h) $(f(x^2))' = f'(x^2)\cdot 2x \longrightarrow f'(2^2)\cdot 2\cdot 2 = 3\cdot 4 = 12$

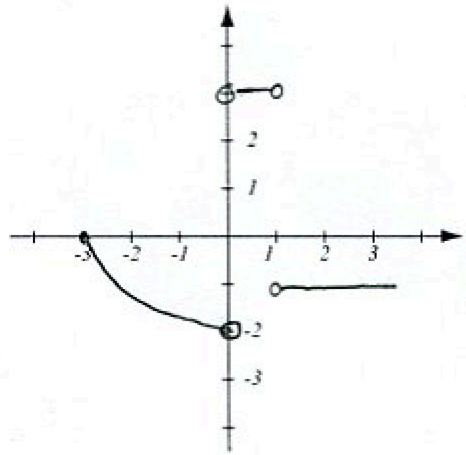
when $x=2$

7. (10pts) The graph of the function $f(x)$ is shown at right.

a) Where is $f(x)$ not differentiable?
b) Use the graph of $f(x)$ to draw an accurate graph of $f'(x)$.



a) Not diff. at $x=0$ (sharp pt)
$x=1$ (not even cont. there)



8. (15pts) Let $f(x) = 3x^2 + 5x - 1$.
a) Use the limit definition of the derivative to find the derivative of the function.
b) Check your answer by taking the derivative of f using differentiation rules.
c) Write the equation of the tangent line to the curve $y = f(x)$ at point $(1, 7)$.

a) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)^2+5(x+h)-1-(3x^2+5x-1)}{h}$

$= \lim_{h\to 0} \frac{3(\cancel{x^2}+2xh+h^2)+\cancel{5x}+5h\cancel{-1}-\cancel{3x^2}-\cancel{5x}+\cancel{1}}{h} = \lim_{h\to 0} \frac{6xh+3h^2+5h}{h}$

$= \lim_{h\to 0} \frac{\cancel{h}(6x+3h+5)}{\cancel{h}} = 6x+5$

b) $(3x^2+5x-1)' = 6x+5$

c) $f'(1) = 11$

$y-7 = 11(x-1)$ $\qquad$ $y = 11x-4$

$y = 11x-11+7$

9. (9pts) A snowball is thrown upwards from ground level with initial velocity 20m/s. Its position is given by the formula $s(t) = -5t^2 + 20t$.
a) Write the formula for the velocity of the snowball at time t.
b) When does the snowball reach its maximum height and what is it?

a) $v(t) = -10t + 20$

b) Max. height occurs when $v(t) = 0$

$-10t + 20 = 0$

$10t = 20$

$t = 2$

$s(2) = -5 \cdot 4 + 20 \cdot 2 = 20$ meters

(max. height)

10. (12pts) Use implicit differentiation to find y'.

$x^2 + y^2 = \sin x \cos y \quad \Big| \frac{d}{dx}$

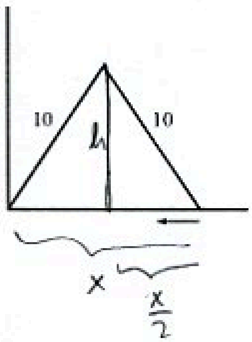
$2x + 2yy' = \cos x \cos y + \sin x(-\sin y)y'$

$2yy' + \sin x \sin y \, y' = \cos x \cos y - 2x$

$y'(2y + \sin x \sin y) = \cos x \cos y - 2x$

$$y' = \frac{\cos x \cos y - 2x}{2y + \sin x \sin y}$$

11. (16pts) A folding ladder whose sides are 10ft long has one end against a wall. If the other end is pushed toward the wall at rate 1/4 foot per second, how fast is the top of the ladder rising when the pushed end is 6 feet away from the wall?

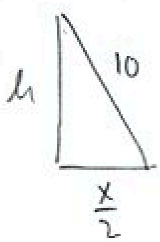


Know: $x' = -\frac{1}{4}$ ft/s

Need: h', when $x = 6$

$$h^2 + \left(\frac{x}{2}\right)^2 = 10^2$$

$$h^2 + \frac{x^2}{4} = 100 \quad \Big| \ \frac{d}{dt}$$



$$2hh' + \frac{2xx'}{4} = 0$$

$$2hh' = -\frac{xx'}{2}$$

$$h' = -\frac{xx'}{4h}$$

When $x = 6$:

$$h^2 + \frac{36}{4} = 100$$

$$h^2 = 91$$

$$h = \pm\sqrt{91} = \sqrt{91} \quad \text{since } h > 0$$

$$h' = -\frac{\overset{3}{\cancel{6}} \cdot \left(-\frac{1}{4}\right)}{\underset{2}{\cancel{4}} \cdot \sqrt{91}} = \frac{3}{8\sqrt{91}} \text{ ft/s}$$

Bonus. (10pts) The Energizer Bunny moves along a straight road so that his position function is $s(t) = t^3 - 15t^2 + 48t + 2$.
a) Find the velocity and acceleration functions and sketch their graphs.
b) When is the Bunny moving forward? Backward?
c) Use the information you found above to sketch the Bunny's path.
d) What is his velocity when acceleration is 0?

$s(2) = 8 - 60 + 96 + 2 = 46$
$s(8) = ((8-15)8+48)8+2 = -62$

a) $v(t) = s'(t) = 3t^2 - 30t + 48$

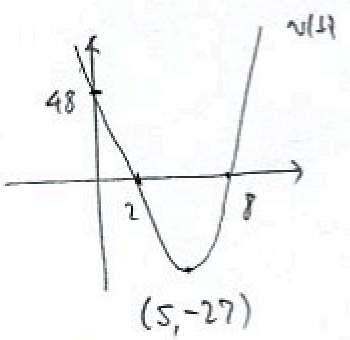
$a(t) = v'(t) = 6t - 30$

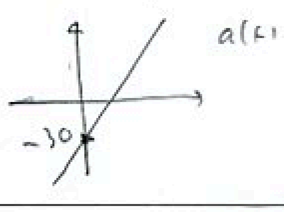
$3t^2 - 30t + 48 = 0$

$3(t^2 - 10t + 16) = 0$

$3(t-2)(t-8) = 0$

$t = 2, 8$

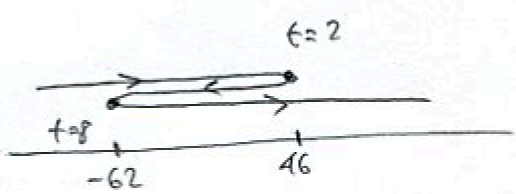




b) Forward $\Leftrightarrow$ when $v(t) > 0$, on $(-\infty, 2)$, $\cup (8, \infty)$

Backward $\Leftrightarrow$ when $v(t) < 0$, on $(2, 8)$

c)



d) $a(t) = 0$

$6t - 30 = 0$

$t = 5$

$v(5) = 3 \cdot 25 - 150 + 48$

$= -27$