Calculus 1 — Exam 1
MAT 250, Spring 2015 — D. Ivanšić

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Show all your work!

1. (16pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$\lim_{x\to -3} f(x) =$ 1

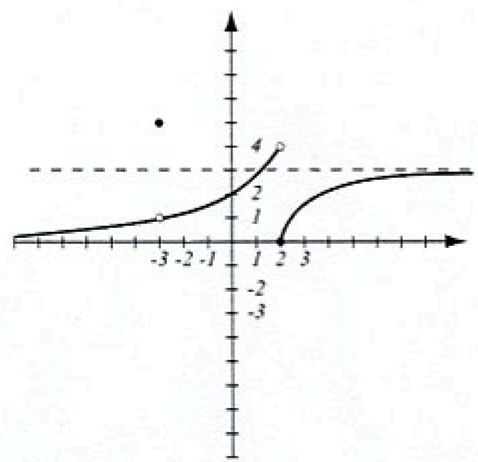
$\lim_{x\to 2^+} f(x) =$ 0

$\lim_{x\to 2^-} f(x) =$ 4

$\lim_{x\to 2} f(x) =$ d.n.e, one-sided limits are not equal

$\lim_{x\to \infty} f(x) =$ 3

$\lim_{x\to -\infty} f(x) =$ 0



List points where f is not continuous and justify why it is not continuous at those points.

f is not continuous at

$x=-3$, here $f(-3) \neq \lim_{x\to -3} f(x)$ $(5 \neq 1)$

$x=2$, $\lim_{x\to 2} f(x)$ d.n.e

2. (4pts) Briefly explain why the function $f(x) = \dfrac{x+3}{3x-2}$ is continuous on its domain.

Linear functions are continuous, and f is a quotient of two continuous functions.

3. (10pts) Find $\lim_{x\to 0} x^4\left(\left(7+\cos\left(\frac{1}{x^3}\right)\right)\right)$. Use the theorem that rhymes with what a doctor may cure.

$$-1 \leq \cos\frac{1}{x^3} \leq 1$$

$$6 \leq 7+\cos\frac{1}{x^3} \leq 8 \quad | \cdot x^4$$

$$6x^4 \leq x^4\left(7+\cos\frac{1}{x^3}\right) \leq 8x^4$$

$$\left.\begin{aligned}\lim_{x\to 0} 6x^4 = 0\\ \lim_{x\to 0} 8x^4 = 0\end{aligned}\right\}$$ equal, so by squeeze theorem,

$$\lim_{x\to 0} x^4\left(7+\cos\frac{1}{x^3}\right) = 0$$

Find the following limits algebraically. Do not use the calculator.

4. (5pts) $\lim_{x\to 7} \frac{x-7}{x^2-2x-35} =$ $\lim_{x\to 7} \frac{\cancel{x-7}}{(x+5)\cancel{(x-7)}} = \lim_{x\to 7} \frac{1}{x+5} = \frac{1}{7+5} = \frac{1}{12}$

5. (7pts) $\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} =$ $\lim_{x\to 16} \frac{\sqrt{x}-4}{x-16} \cdot \frac{\sqrt{x}+4}{\sqrt{x}+4} = \lim_{x\to 16} \frac{(\sqrt{x})^2-4^2}{(x-16)(\sqrt{x}+4)}$

$= \lim_{x\to 16} \frac{\cancel{x-16}}{\cancel{(x-16)}(\sqrt{x}+4)} = \lim_{x\to 16} \frac{1}{\sqrt{x}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{8}$

6. (7pts) $\lim_{x\to 0} \frac{\sin(3x)}{5x} =$ $\lim_{x\to 0} \frac{\sin(3x)}{5x} \cdot \frac{3}{3} = \lim_{x\to 0} \underbrace{\frac{\sin(3x)}{3x}}_{\to 1} \cdot \frac{3}{5} = 1 \cdot \frac{3}{5} = \frac{3}{5}$

7. (7pts) $\lim_{x\to\infty} \frac{4x^3-3x^2-7x+2}{7x^3-x^2+5x} =$ $\lim_{x\to\infty} \frac{\cancel{x^3}(4-\overbrace{\frac{3}{x}}^{\to 0}-\overbrace{\frac{7}{x^2}}^{\to 0}+\overbrace{\frac{2}{x^3}}^{\to 0})}{\cancel{x^3}(7-\underbrace{\frac{1}{x}}_{\to 0}+\underbrace{\frac{5}{x^2}}_{\to 0})} = \frac{4-0-0+0}{7-0+0} = \frac{4}{7}$

8. (6pts) $\lim_{x\to 5^+} \frac{4-2x}{x-5} =$ $\frac{-6}{0+} = -\infty$

When $x>5$
$x-5>0$

$\left(\text{or } \frac{-6}{\text{small pos}} = \text{large neg.}\right)$

9. (10pts) Use the Intermediate Value Theorem to show that the equation $x^3+2x = 4\sqrt{x}+2$ has at least one solution.

Rewrite as $x^3+2x-4\sqrt{x}-2=0$

Let $f(x)= x^3+2x-4\sqrt{x}-2$, it is continuous.

$f(0)=-2$

$f(2)= 8+4-4\sqrt{2}-2 = 10-4\sqrt{2}=4.34..$

Since 0 is between -2 and 4.34, by IVT there is a c in $(0,2)$ such that $f(c)=0$.

10. (10pts) Consider the limit $\lim_{x\to 0} \frac{5^x-1}{x}$. Use your calculator to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.

x	$\frac{5^x-1}{x}$	x	$\frac{5^x-1}{x}$
0.01	1.622459	-0.01	1.596556
10^{-3}	1.610734	-10^{-3}	1.608143
10^{-4}	1.609567	-10^{-4}	1.609308
10^{-5}	1.609451	-10^{-5}	1.609425
10^{-6}	1.609439	-10^{-6}	1.609436

It appears the limit is approximately 1.6094

11. (4pts) Consider the limit below, representing a derivative $f'(a)$: find f and a.

$\lim_{h\to 0} \frac{(3+h)^4-81}{h} = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$f(x)=x^4$

$a=3$

$(f(3)= 3^4=81)$

12. (14pts) The amount of water (in gallons) in a 100-gallon tank that is draining at the bottom is given by $V(t) = t^2 - 20t + 100$, where t is in minutes, $0 \le t \le 10$.
a) What is the average rate of draining from $t = 2$ to $t = 5$? What are the units?
b) What is the instantaneous rate of draining when $t = 2$? What are the units?

a) $\frac{V(5)-V(2)}{5-2} = \frac{25-64}{3} = \frac{-39}{3} = -13$ gallons/minute

b) $\lim_{t\to 2} \frac{V(t)-V(2)}{t-2} = \lim_{t\to 2} \frac{t^2-20t+100-64}{t-2} = \lim_{t\to 2} \frac{t^2-20t+36}{t-2}$

$= \lim_{t\to 2} \frac{(t-18)\cancel{(t-2)}}{\cancel{t-2}} = \lim_{t\to 2} (t-18) = 2-18 = -16$ gallons/minute

Bonus. (10pts) Consider the limit $\lim_{x\to\sqrt{3}} \frac{x^2-3}{\cancel{\sqrt{x}}\,x-\sqrt{3}}$.
a) Use your calculator to estimate this limit with accuracy 4 decimal points. Write a table of values that will justify your answer.
b) Find the limit algebraically.

$\sqrt{3} \approx 1.732051$

x	$\frac{x^2-3}{x-\sqrt{3}}$
1.73	3.462051
1.732	3.464051
1.73205	3.464101
1.732051	3.464102
1.72	3.452051
1.731	3.463051
1.7319	3.463951
1.73204	3.464091

It appears lim is 3.4641

$\lim_{x\to\sqrt{3}} \frac{x^2-3}{x-\sqrt{3}} = \lim_{x\to\sqrt{3}} \frac{(x-\sqrt{3})(x+\sqrt{3})}{x-\sqrt{3}}$

$= \lim_{x\to\sqrt{3}} (x+\sqrt{3}) = 2\sqrt{3} \approx 3.464102$