1. (6pts) Draw arrow diagrams between two sets that illustrate

a) a surjection   b) an injection that is not a surjection   c) something that is not a function

2. (14pts) Let $\mathbf{Z}_6 = \{0, 1, 2, 3, 4, 5\}$, and let $f : \mathbf{Z}_6 \to \mathbf{Z}_6$, $f(x) = 2x + 1 \, (\mathrm{mod} \, 6)$.
a) Write the table of function values.
b) What is the set of preimages of 3?
c) What is the set of preimages of 4?
d) Is f injective? Justify.
e) Is f surjective? Justify.

3. (16pts) Use induction to prove: for every $n \in \mathbf{N}$ and every $x \in \mathbf{R}$, $x \neq 1$,

$$1 + x + x^2 + x^3 + \cdots + x^n = \frac{x^{n+1} - 1}{x - 1}.$$

4. (18pts) Let $g : \mathbf{Z} \times \mathbf{Z} \to \mathbf{Z} \times \mathbf{Z}$ be given by $g(m, n) = (2m + n, 5m + 3n)$.
a) Evaluate $g(3, -4)$ and $g(-1, -3)$.
b) Find the set of preimages of $(-1, 6)$.
c) Show that g is surjective.

5. (18pts) Let $f(x) = \dfrac{4}{x^2 + 1}$. We assume the codomain is $\mathbf{R}$.

a) What subset of real numbers is the natural domain for this function?

b) Is this function injective? Justify.

c) What is the range of this function? Justify your answer.

6. (8pts) Let $h : \mathbf{R} - \{\frac{1}{3}\} \to \mathbf{R}$ be given by $h(x) = \dfrac{2x}{3x - 1}$. Show that h is injective.

7. (20pts) Use induction to prove: when dividing by 3 the sum of squares of three consecutive natural numbers one always gets remainder 2. (For example, dividing $7^2 + 8^2 + 9^2$ by 3 gives remainder 2.)

Bonus. (10pts) Construct a bijection $\mathbf{N} \to \mathbf{Z}$. Use an arrow diagram to get an idea.