

Mathematical Reasoning — Exam 2
MAT 312, Fall 2011 — D. Ivanišić

Name: _____
Show all your work!

1. (12pts) Let U be the set of real numbers. Consider the intervals $A = [-3, 2]$, $B = [-1, 0]$, $C = (0, \infty)$ and write the following subsets in interval notation (draw the real line if it helps):

$A \cap C$ $A \cup C$ $A - B$ $B - C$ A^c $A \cap B \cap C$

2. (18pts) Let A , B and C be subsets of some universal set U .

a) Use Venn diagrams to draw the following subsets.

b) Among the four sets, two are equal. Use set algebra to show they are equal.

$(A - B) - C$ $A - (B - C)$ $A - (B \cap C)$ $A - (B \cup C)$

3. (14pts) Let $A = \{x \in \mathbf{Z} \mid x \equiv 2 \pmod{3}\}$ and $B = \{x \in \mathbf{Z} \mid x \equiv 5 \pmod{6}\}$.
- a) Is $A \subseteq B$? Prove or disprove.
 - b) Is $B \subseteq A$? Prove or disprove.

4. (10pts) Prove: for every real number x , if x is irrational, then $\frac{1}{x}$ is irrational.

5. (14pts) Let A, B be subsets of a universal set U . Prove that $A = B$ if and only if $A \cup B = A \cap B$. (Note: one direction can be done simply by set algebra.)

6. (18pts) Prove the following:

- a) For every integer a , if a^3 is divisible by 3, then a is divisible by 3.
- b) $\sqrt[3]{9}$ is an irrational number. (Use statement a)).

7. (14pts) Prove that for every real number $a \geq 0$, $a + \frac{1}{a} \geq 2$.

Bonus. (10pts) Prove that 131,739,418 is not a square of any integer.