Mathematical Reasoning — Exam 1
MAT 312, Fall 2011 — D. Ivanšić

Name: _____

Show all your work!

1. (17pts) Consider the following sentences. If a sentence is a statement, determine whether it is true (and justify your answer). If it is a predicate, find its truth set.

a) (universal set=$\mathbf{R}$) $x^2 - 5x + 6 = 0$

b) If an integer is divisible by 4, then it is divisible by 6.

c) If pigs fly, then the sun sets in the east.

d) There exists a real number x such that $x^2 - 7 = 0$.

e) For every $x \in \mathbf{R}$, if $x > 1$, then $x^4 > x^2$.

2. (8pts) Negate the following statements.

a) Leia does not answer the questions and she suffers the consequences.

b) If Luke goes to the Dagobah system, he gets stuck in the swamp or masters the force.

3. (8pts) Use a truth table to prove the equivalence $P \iff Q \equiv (\neg P \vee Q) \wedge (P \vee \neg Q)$. (Use however many columns you need.)

P	Q								
T	T								
T	F								
F	T								
F	F								

4. (12pts) Use previously proven logical equivalences to prove the equivalence $P \implies (Q \implies R) \equiv (P \wedge Q) \implies R$. Do not use a truth table.

5. (4pts) Write the converse and contrapositive of the statement: If $x > 0$, then $3^x > 1$.

Converse:

Contrapositive:

6. (4pts) Use the roster method to write the set $\{x \in \mathbf{Z} \mid x^2 + 4 < 10\}$.

7. (6pts) Use set builder notation to write the set $\{\ldots, -5, -1, 3, 7, 11, \ldots\}$

8. (14pts) For each of the following statements, do the following:
a) Write the statement using symbols.
b) Write the negation of the statement using symbols.
c) Write the negation of the statement in English.

1) There exist real numbers x and y such that $\sin x + \sin y = 3$.

2) There exists a positive real number y such that for every real number x, if $x^2 + y < 7$, then $x^2 + y^2 > 25$.

9. (12pts) Let $\mathbf{R}$ be the universal set. The following is a predicate in x:

$$(\exists y \in \mathbf{R})(x^2 - y^2 = 16)$$

a) If $x = 3$, is the statement true?
b) If $x = 7$, is the statement true?
c) Find the truth set (the x's) of the above statement. Write it using interval notation.

10. (15pts) An integer n is called a type-0, type-1 or type-2 integer if it can be written in the form $n = 3k$, $n = 3k + 1$ or $n = 3k + 2$, respectively, for some integer k. Prove that if m is a type-1 integer and n is a type-2 integer, then $m^2 + mn + n^2$ is a type-1 integer. Start with a know-show table if you find it helpful.

Bonus. (10pts) Determine whether the statements 1 and 2 in problem 8 are true and justify.