Simplify and write the answer so all exponents are positive:

1. (5pts) $(3x^4y^{-2})^2(5x^{-2}y^6)^3 =$

2. (7pts) $\dfrac{(4a^3b^{-4})^3}{(12a^3b^5)^2} =$

3. (4pts) Convert to scientific notation or a decimal number:

$32,987,041 =$ $1.4783 \times 10^{-3} =$

Simplify and write in standard form:

4. (4pts) $(x^2+3)5x - (2x-3)(x-7) =$

5. (4pts) $(2x-1)(3x^2+4x-2) =$

Use formulas to expand:

6. (4pts) $(3x - 5y)(3x + 5y) =$

7. (5pts) $(4x^3 - 7y)^2 =$

8. (6pts) $(5x + 4)^3 =$

Factor the following. Use either a known formula or a factoring method.

9. (3pts) $x^2 + 3x - 18 =$

10. (6pts) $8x^2 + 2x - 15 =$

11. (6pts) $27w^3 - 125 =$

12. (6pts) The population of Hong Kong was 7,071,576 according to 2011 census data. If its land area is 1104 square kilometers, find the population density (people per square kilometer) and write it in scientific notation, rounded to six decimal points.

Solve the equations.

1. (3pts) $4x - 9 = 2x - 3 + 5(x - 1)$

2. (6pts) $x^2 - x + 4 = 3x + 25$

3. (5pts) Solve the equation for y:

$L = ax - by^2$

Simplify.

4. (8pts) $\dfrac{2x}{3x^2 + 11x - 4} - \dfrac{x}{x^2 + 6x + 8} =$

5. (8pts) $\dfrac{\dfrac{x}{2} - \dfrac{1}{3}}{\dfrac{5}{6x} + 1} =$

Simplify, showing intermediate steps. Assume variables can be any real numbers.

6. (2pts) $\sqrt{162} =$

7. (4pts) $\sqrt{28x^5y^6} =$

8. (5pts) $\dfrac{\sqrt{45x^3y^7}}{\sqrt{20xy^4}} =$

9. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to root notation.

$$\dfrac{\left(x^{\frac{1}{3}}y^{-\frac{3}{4}}\right)^2}{\left(x^4y^{\frac{1}{2}}\right)^{\frac{1}{2}}} =$$

10. (6pts) Simplify.

$(\sqrt{3} + 2\sqrt{5})(3\sqrt{5} - 4\sqrt{3}) =$

**11.** (5pts) Rationalize the denominator.

$\dfrac{\sqrt{7} - 2}{\sqrt{7} + 5}$

1. (8pts) Draw the triangle with vertices $A = (-1, 4)$, $B = (-2, 0)$ and $C = (5, 2)$. Then use the distance formula and the Pythagorean theorem to find out whether the triangle is a right triangle

2. (8pts) Write the equation of the circle whose diameter has endpoints $(-3, -2)$ and $(1, 2)$. Sketch the circle.

3. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(3)$ and $f(-3)$.

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions
   of the equation $f(x) = 4$?
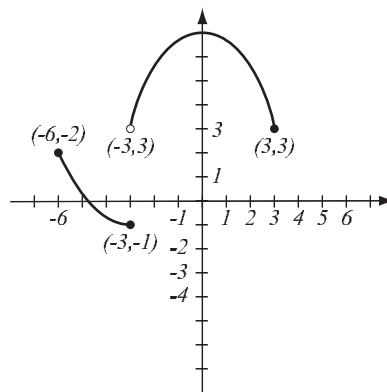
4. (12pts) The function
$f(x) = x\sqrt{x+5} - 3$ is given.

a) Use your calculator to accurately
its graph. Draw the graph here, and
indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts
(accuracy: 6 decimal points).

c) State the domain and range.

5. (12pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{\sqrt{x}}{4x - 11}$$

$$g(x) = \frac{3x + 7}{x^2 - 3x - 10}$$

6. (12pts) Let $g(x) = \frac{x^2 + 3x - 1}{4x - 2}$. Find the following (simplify where appropriate).

$g(-1)$

$g\left(\frac{1}{2}\right)$

$g(3u)$

$g(x + 4)$

1. (6pts) Find the equation of the line (in form $y = mx + b$) that has x-intercept -1 and y-intercept 2.

2. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through point $(-1, -2)$ and is perpendicular to the line with slope -2 that goes through the point $(3, -2)$. Draw both lines.

3. (8pts) Draw the lines $2x - 3y = 6$, $2x - 3y = 12$, $7x + 5y = 10$ and $7x + 5y = 30$. Use slopes to determine if the quadrilateral that these lines form is a
a) parallelogram   b) rectangle.

4. (4pts) To cut costs, many corporations have been selling their private jets. The number of used jets for sale has increased from 1022 in 1999 to 3014 in 2009. What is the average rate of change in the number of used jets for sale from 1999 to 2009? What are the units for the average rate?

5. (12pts) Antonio is the customer of a electric company. In one month he used 1220 kilowatt-hours and paid \$149.90. In another month he used 2358 kilowatt-hours and paid \$275.08.

a) Assuming that monthly cost $C(x)$ is a linear function of kilowatt-hours used x, write a formula for $C(x)$.

b) How much does Antonio pay if he uses 0 kilowatt-hours? What is the meaning of this number?

c) What is the meaning of the slope in this example?

6. (20pts) A researcher is studying the number of chirps crickets emit in a minute and the ambient temperature in order to model the relationship between the number of chirps and the temperature. The table shows the data: n is the number of chirps per minute, and T is the temperature in °F. Solve the problems below with accuracy 6 decimal points.

a) Draw the scatterplot of the data. Does the relationship look linear?

b) Use two points in the scatterplot to get an equation of a line that models the relationship between n and T. Draw the line on the graph.

c) Use your calculator to find the "line of best fit" for the data. Draw the line on the graph.

d) Find the coefficient of correlation r. How strong is the linear relationship between n and T?

e) What temperature does the researcher expect if crickets chirp 125 times per minute?

n	T
75	55
82	59
92	61
101	64
115	64
140	70
157	77

Solve the inequalities. Write your solution in interval notation.

1. (5pts) $-2 \leq 3x - 4 < 8$

2. (7pts) $2x - 2 \leq -1$ or $4x + 3 > 12$

3. (6pts) Find the domain of the function $f(x) = \dfrac{\sqrt{3x + 11}}{2x - 7}$ (in interval notation).

4. (14pts) You have \$20,000 to invest and can split this money between an investment bringing 5% interest, and one bringing 6.5% interest. What is the least you need to invest at 6.5% interest in order to meet a goal of annual interest of at least \$1,100?

5. (14pts) Max rows his boat at 6mph in still water. One day he takes a round-trip on a river, taking 2 hours to row downstream, and then 3 hours to row back upstream. What is the speed of the river? How far did Max travel in one direction?

6. (14pts) How many liters of pure water must be mixed with 4 liters of a 20% solution of sulphuric acid in order to get a 13% solution of sulphuric acid?

1. (10pts) Use your calculator to accurately sketch the graph of the function
$f(x) = x^2(x-5)^{\frac{2}{3}}$. Draw the graph here, and indicate units on the axes.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let $f(x) = \sqrt{3-2x}$, $g(x) = \dfrac{x^2}{x^2-9}$. Find the following (simplify where possible):

$(f+g)(1) =$ $(fg)(-1) =$

$\dfrac{f}{g}(x) =$ $(f \circ g)(2) =$

$(g \circ f)(x) =$

The domain of $(f-g)(x)$ in interval notation

3. (8pts) Consider the function $h(x) = \dfrac{\sqrt{x+1}}{4}$. Find functions f and g so that $h(x) = f(g(x))$. Find two different solutions to this problem, neither of which is the "stupid" one.

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x + 1, & \text{if } x \leq -2 \\ x - 5, & \text{if } -2 < x \leq 6. \end{cases}$$

5. (14pts) A farmer's market wishes to build a block of 10 stalls separated by walls (see picture). The market has enough money for 250 feet of walls and wishes to maximize the area of the block.

a) Express the area of the block as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the maximum. What are the dimensions of the block that give the maximum area?
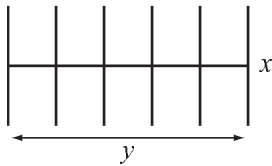
1. (21pts) For the following functions, determine algebraically whether they odd, even, or neither. Then use the calculator to draw their graphs on paper and verify your conclusions.

$f(x) = x^3 + x^2$ $\qquad\qquad$ $g(x) = 2|x| - 2$ $\qquad\qquad$ $h(x) = x^3 - x$

2. (16pts) Using transformations, draw the graphs of $f(x) = 2|x+3|$ and $g(x) = -\left(\frac{2}{3}x\right)^2 + 4$. Explain how you transform graphs of basic functions in order to get the graphs of f and g.

3. (9pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^2$, shifted left 3 units
b) shape of $y = \frac{1}{x}$, stretched horizontally by factor 4, then shifted up 1 unit
c) shape of $y = \sqrt[3]{x}$, stretched vertically by factor 2, then reflected about the y-axis, then shifted right 5 units.

4. (14pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x) + 3$, $f\left(\frac{1}{2}x\right)$ and $-f(-x)$ and label all the relevant points.
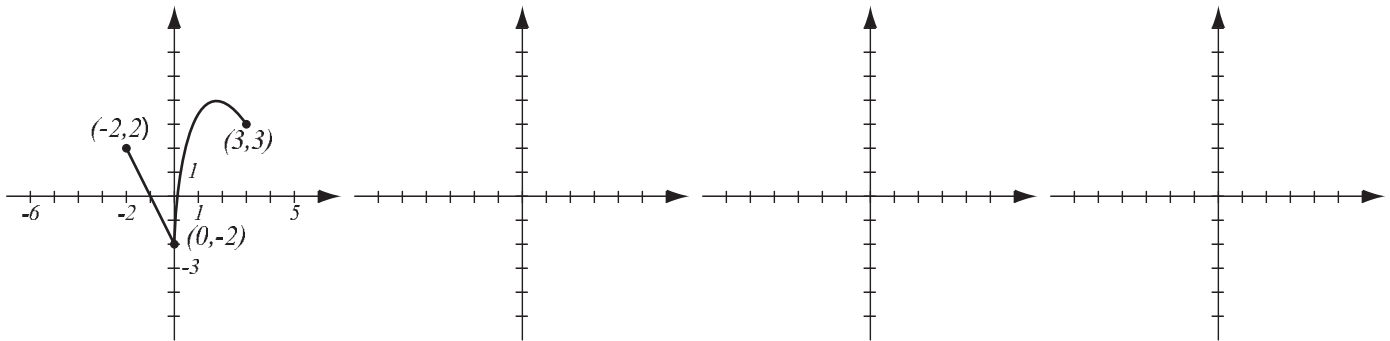
Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(5 - i)(3 - 2i) =$

2. (6pts) $\dfrac{5 + 3i}{2 - 7i} =$

3. (4pts) Simplify and justify your answer.

$i^{217} =$

4. (6pts) Starting from rest, an object falling t seconds travels approximately $s = 5t^2$ meters. How long would it take for a piano to fall from a tower 90 meters tall?

5. (8pts) Solve the equation: $2x^4 + 5x^2 - 63 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 10x + 31 = 0$

7. (12pts) The quadratic function $f(x) = -x^2 - x + 6$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

8. (14pts) Farmer Andrew has a rectangular plot of land of length 400 and width 300 feet. Due to rising demand for his products, he plans to clear the adjoining forest in order to increase both length and width to get a bigger rectangular plot of 200,000 square feet. If width is increased by an amount that is twice as much as the increase in length, by how much are length and width increased to get a plot of desired area?

1. (4pts) Solve the equation.

$|7x - 2| = 30$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|x + 3| \geq 4$ $|3x - 5| < 7$

Solve the equations:

3. (8pts) $\dfrac{x}{x+1} + \dfrac{10}{x+3} = \dfrac{2x^2 + 9x - 11}{x^2 + 4x + 3}$ **4.** (8pts) $\sqrt{x + 45} - x = 3$

5. (14pts) A ball is thrown upwards from the ground with initial velocity 32 meters per second. Its height in meters after t seconds is given by $s(t) = -5t^2 + 32t$.
a) Sketch the graph of the height function.
b) When does the ball reach its greatest height, and what is that height?
c) When is the ball at height 44 meters?

6. (14pts) You are building a simple rectangular building with two rooms and a 10-ft opening between them and have enough money to build 300 feet of walls (see picture). Your goal is to maximize the enclosed area.
a) Express the area of the building as a function of one of the sides of the rectangle. What is the domain of this function?
c) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the greatest area and what is the greatest area possible?
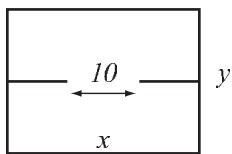
1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.

(4,6)
6
3
2
1
-6   *-3 -2 -1*   *1 2*   *4 5*
(-1,-4) *-3*   *(2,-2)*
-5
(-6,-5)
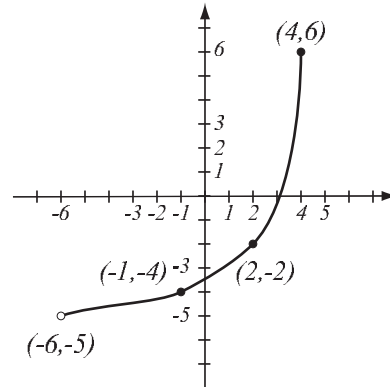
2. (12pts) Let $f(x) = \dfrac{3x+2}{5x-3}$. Find the formula for f^{-1}. Find the domain and range of f.

3. (8pts) Evaluate without using the calculator:

$\log_3 243 =$ $\log_2 \frac{1}{64} =$ $\log_9 27 =$ $\log_b \sqrt[4]{b^9} =$

4. (4pts) Use your calculator to find $\log_6 0.82$ with accuracy 6 decimal places. Show how you obtained your number.

5. (5pts) If you invest \$4,000 in an account bearing 4.11%, compounded monthly, how much is in the account in 31 months?

6. (3pts) Find the domain of $f(x) = \log_2(5 - 7x)$.

7. (7pts) The population of Nashville (in thousands) may be described by the function $P(t) = 569(1.00975)^t$, where t is the number of years since 2000.
a) Find the estimated populations for 2010 and the predicted population for 2018.
b) According to the model, when will the population of Nashville reach 700 thousand?

Solve the equations.

8. (3pts) $5 = e^{x^2}$ **9.** (5pts) $5^{x-3} = \left(\dfrac{1}{25}\right)^{2x-1}$ **10.** (7pts) $3^{x-2} = 4^{2x-3}$