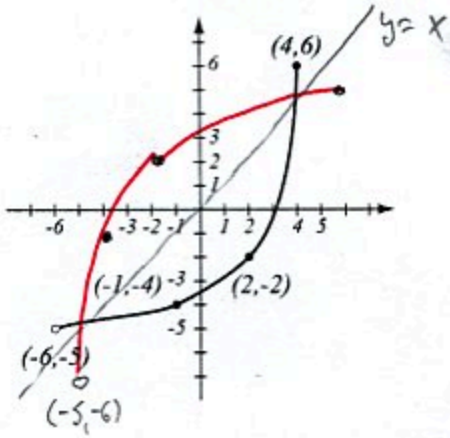
College Algebra — Joysheet 10
MAT 140, Fall 2015 — D. Ivanšić

Name:

Covers: 5.1–5.5 *Show all your work!*

1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) yes - it passes the horizontal line test

b) see graph

2. (12pts) Let $f(x) = \dfrac{3x+2}{5x-3}$. Find the formula for f^{-1}. Find the domain and range of f.

$y = \dfrac{3x+2}{5x-3}$

$(5x-3)y = 3x+2$

$5xy - 3y = 3x+2$

$5xy - 3x = 3y + 2$

$x(5y-3) = 3y+2$

$x = \dfrac{3y+2}{5y-3}$ $\quad f^{-1}(y) = \dfrac{3y+2}{5y-3}$

Domain of f: Can't have

$5x-3=0$

$5x=3$

$x=\frac{3}{5}$

$(-\infty, \frac{3}{5}) \cup (\frac{3}{5}, \infty)$

Range of f = Domain of f^{-1}: can't have

$5y-3=0$

$5y=3$

$y=\frac{3}{5}$

$(-\infty, \frac{3}{5}) \cup (\frac{3}{5}, \infty)$

3. (8pts) Evaluate without using the calculator:

$\log_3 243 = 5$ $\quad 3^? = 243$

$\log_2 \frac{1}{64} = -6$ $\quad 2^? = \frac{1}{64} = \frac{1}{2^6} = 2^{-6}$

$\log_9 27 = \frac{3}{2}$ $\quad 9^? = 27 = 3^3 = \sqrt{9}^3 = 9^{\frac{3}{2}}$

$\log_b \sqrt[4]{b^9} = \frac{9}{4}$ $\quad b^? = \sqrt[4]{b^9} = b^{\frac{9}{4}}$

4. (4pts) Use your calculator to find $\log_6 0.82$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_6 0.82 = \dfrac{\ln 0.82}{\ln 6} = -0.110758$

5. (5pts) If you invest \$4,000 in an account bearing 4.11%, compounded monthly, how much is in the account in 31 months?

$A = P\left(1+\frac{r}{n}\right)^{nt}$ 31 months $= \frac{31}{12}$ years

$A = 4000\left(1+\frac{0.0411}{12}\right)^{12\cdot\frac{31}{12}} = 4000(1.003425)^{31} = 4000\cdot 1.11\ldots = 4447.26$

6. (3pts) Find the domain of $f(x) = \log_2(5-7x)$.

Must have $5-7x>0$

$5>7x$

$x<\frac{5}{7}$ $\quad (-\infty, \frac{5}{7})$

7. (7pts) The population of Nashville (in thousands) may be described by the function $P(t) = 569(1.00975)^t$, where t is the number of years since 2000.

a) Find the estimated populations for 2010 and the predicted population for 2018.

b) According to the model, when will the population of Nashville reach 700 thousand?

a) $P(10) = 569\cdot 1.00975^{10}$

$= 626.975954$

$P(18) = 569\cdot 1.00975^{18}$

$= 677.581874$

b) $569\cdot 1.00975^t = 700$

$1.00975^t = \frac{700}{569}$ | ln

$\ln(1.00975^t) = \ln\frac{700}{569}$

$t\ln 1.00975 = \ln\frac{700}{569}$

$t = \frac{\ln\frac{700}{569}}{1.00975} = 21.354704$

ln approx. 21.35 yrs, so, in 2022

Solve the equations.

8. (3pts) $5 = e^{x^2}$

$\ln 5 = \ln e^{x^2}$

$\ln 5 = x^2$

$x = \pm\sqrt{\ln 5}$

$= \pm 1.268636$

9. (5pts) $5^{x-3} = \left(\frac{1}{25}\right)^{2x-1}$

$5^{x-3} = (5^{-2})^{2x-1}$

$5^{x-3} = 5^{-4x+2}$

$x-3 = -4x+2$

$5x = 5$

$x = 1$

10. (7pts) $3^{x-2} = 4^{2x-3}$ | ln

$\ln 3^{x-2} = \ln 4^{2x-3}$

$(x-2)\ln 3 = (2x-3)\ln 4$

$x\ln 3 - 2\ln 3 = 2x\ln 4 - 3\ln 4$

$3\ln 4 - 2\ln 3 = 2x\ln 4 - x\ln 3$

$x(2\ln 4 - \ln 3) = 3\ln 4 - 2\ln 3$

$x = \frac{3\ln 4 - 2\ln 3}{2\ln 4 - \ln 3} = 1.171856$