College Algebra — Joysheet 8
MAT 140, Fall 2015 — D. Ivanšić

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Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(5 - i)(3 - 2i) = 15 - 10i - 3i + \underbrace{2i^2}_{-2} = 13 - 13i$

2. (6pts) $\frac{5 + 3i}{2 - 7i} = \frac{5+3i}{2-7i} \cdot \frac{2+7i}{2+7i} = \frac{10+35i+6i+21i^2}{2^2-(7i)^2} = \frac{10+41i-21}{4-(-49)}$

$= \frac{-11+41i}{53}$

3. (4pts) Simplify and justify your answer.

$i^{217} = i^{216} \cdot i^1 = \underbrace{i^{4 \cdot 54}}_{(i^4)^{54} = 1^{54} = 1} \cdot i = i$

4. (6pts) Starting from rest, an object falling t seconds travels approximately $s = 5t^2$ meters. How long would it take for a piano to fall from a tower 90 meters tall?

$5t^2 = 90$

$t^2 = \frac{90}{5} = 18$

$t = \pm\sqrt{18} = \pm 3\sqrt{2}$

$t = 3\sqrt{2}$ since a negative number doesn't fit the context.

5. (8pts) Solve the equation: $2x^4 + 5x^2 - 63 = 0$

Let $u = x^2$

$2u^2 + 5u - 63 = 0$

$u = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 2(-63)}}{2 \cdot 2}$

$u = \frac{-5 \pm \sqrt{25+504}}{4} = \frac{-5 \pm \sqrt{529}}{4} = \frac{-5 \pm 23}{4} = -7, \frac{9}{2}$

$x^2 = -7$ $\quad x^2 = \frac{9}{2}$

$x = \pm\sqrt{7}\, i$ $\quad x = \pm\frac{3}{\sqrt{2}}$

6. (6pts) Solve by completing the square.

$x^2 - 10x + 31 = 0$ $\quad | + \left(\frac{10}{2}\right)^2 = 5^2$

$x^2 - 2 \cdot x \cdot 5 + 5^2 + 31 = 5^2$

$(x-5)^2 = 25 - 31$

$(x-5)^2 = -6$

$x - 5 = \pm\sqrt{6}\, i$

$x = 5 \pm \sqrt{6}\, i$

7. (12pts) The quadratic function $f(x) = -x^2 - x + 6$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any. Find the y-intercept.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) x-int.

$-x^2 - x + 6 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$

y-int: $f(0) = 6$

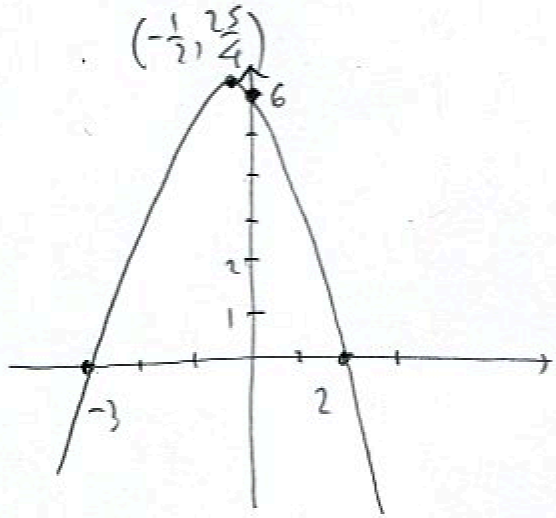
b) $h = -\frac{-1}{2\cdot(-1)} = -\frac{1}{2}$

$f(-\frac{1}{2}) = -(-\frac{1}{2})^2 - (-\frac{1}{2}) + 6$

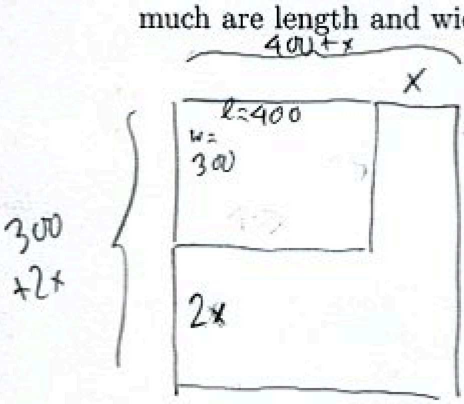
$= -\frac{1}{4} + \frac{1}{2} + 6 = 6 + \frac{1}{4}$

$= 6\frac{1}{4} = \frac{25}{4}$

vertex: $(-\frac{1}{2}, \frac{25}{4})$



8. (14pts) Farmer Andrew has a rectangular plot of land of length 400 and width 300 feet. Due to rising demand for his products, he plans to clear the adjoining forest in order to increase both length and width to get a bigger rectangular plot of 200,000 square feet. If width is increased by an amount that is twice as much as the increase in length, by how much are length and width increased to get a plot of desired area?



Area = 200000

x = increase in length

$(400+x)(300+2x) = 200000$

$120000 + 1100x + 2x^2 = 200000$

$2x^2 + 1100x - 80000 = 0 \quad | \div 2$

$x^2 + 550x - 40000 = 0$

25·185

$$x = \frac{-550 \pm \sqrt{550^2 - 4\cdot 1(-40000)}}{2\cdot 1} = \frac{-550 \pm \sqrt{10^2(55^2 + 1600)}}{2} = \frac{-550 \pm 10\sqrt{4625}}{2}$$

$$= \frac{-550 \pm 10\cdot 5\cdot\sqrt{185}}{2} = -275 \pm 25\sqrt{185}$$

The negative solution $-225 - 50\sqrt{57}$ does not fit context $(x > 0)$

Increase length by $x = -275 + 25\sqrt{185} = 65.036763$

width by $2x = -550 + 50\sqrt{185} = 130.073525$