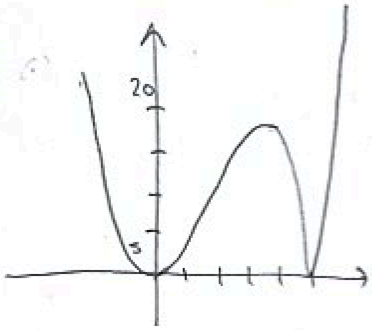
College Algebra — Joysheet 6
MAT 140, Fall 2015 — D. Ivanšić

Name: Saul Ocean

Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = x^2(x-5)^{\frac{2}{3}}$. Draw the graph here, and indicate units on the axes.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.



a) Local minima: $f(0)=0$ and $f(5)=0$

Local maximum: $f(3.75) = 16.318086$

b) Increasing on $(0, 3.75) \cup (5, \infty)$

Decreasing on $(-\infty, 0) \cup (3.75, 5)$

2. (20pts) Let $f(x) = \sqrt{3-2x}$, $g(x) = \dfrac{x^2}{x^2-9}$. Find the following (simplify where possible):

$(f+g)(1) = f(1) + g(1)$

$= \sqrt{3-2\cdot 1} + \dfrac{1^2}{1^2-9} = 1 + \dfrac{1}{-8} = \dfrac{7}{8}$

$(fg)(-1) = f(-1)\cdot g(-1) = \sqrt{3-2(-1)} \cdot \dfrac{(-1)^2}{(-1)^2-9}$

$= \sqrt{5} \cdot \dfrac{1}{-8} = -\dfrac{\sqrt{5}}{8}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\sqrt{3-2x}}{\frac{x^2}{x^2-9}} =$

$= \sqrt{3-2x} \cdot \dfrac{x^2-9}{x^2} = \dfrac{(x^2-9)\sqrt{3-2x}}{x^2}$

$(f\circ g)(2) = f(g(2)) = f\left(\dfrac{2^2}{2^2-9}\right) = f\left(\dfrac{4}{-5}\right)$

$= \sqrt{3-2\cdot\left(-\frac{4}{5}\right)} = \sqrt{3+\frac{8}{5}} = \sqrt{\dfrac{23}{5}}$

$(g\circ f)(x) = g(f(x)) = g\left(\sqrt{3-2x}\right) = \dfrac{\left(\sqrt{3-2x}\right)^2}{\left(\sqrt{3-2x}\right)^2-9} = \dfrac{3-2x}{3-2x-9} = \dfrac{3-2x}{-2x-6}$

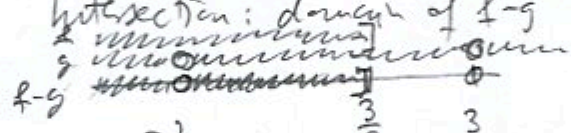
$= \dfrac{2x-3}{2x+6}$

The domain of $(f-g)(x)$ in interval notation

Domain of f: Must have
$3-2x \geq 0$
$2x \leq 3$
$x \leq \frac{3}{2}$
$(-\infty, \frac{3}{2}]$

Domain of g: Can't have
$x^2-9=0$
$x^2=9$
$x=\pm 3$
$(-\infty,-3) \cup (-3,3) \cup (3,\infty)$

Intersection: domain of f-g



$(-\infty, -3) \cup (-3, \frac{3}{2}]$

3. (8pts) Consider the function $h(x) = \dfrac{\sqrt{x+1}}{4}$. Find functions f and g so that $h(x) = f(g(x))$. Find two different solutions to this problem, neither of which is the "stupid" one.

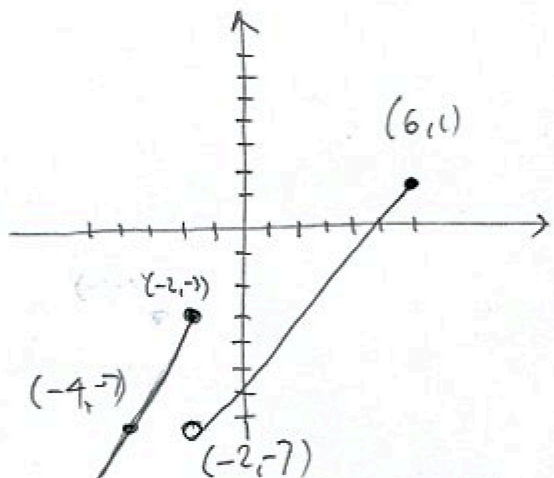
$f(g(x)) = \dfrac{\sqrt{x+1}}{4}$ $\quad g(x) = x+1$, $f(x) = \dfrac{\sqrt{x}}{4}$ $\quad g(x) = \sqrt{x+1}$, $f(x) = \dfrac{x}{4}$

4. (8pts) Sketch the graph of the piecewise-defined function:

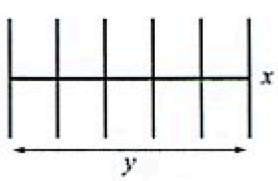
$$f(x) = \begin{cases} 2x+1, & \text{if } x \le -2 \\ x-5, & \text{if } -2 < x \le 6. \end{cases}$$

x	$2x+1$
-2	-3
-4	-7

x	$x-5$
-2	-7
6	1



5. (14pts) A farmer's market wishes to build a block of 10 stalls separated by walls (see picture). The market has enough money for 250 feet of walls and wishes to maximize the area of the block.
a) Express the area of the block as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the maximum. What are the dimensions of the block that give the maximum area?



a) $6x + y = 250$

so $y = 250 - 6x$

$A = xy = x(250 - 6x)$

$A(x) = -6x^2 + 250x$

Domain:
Must have

$x \geq 0$

$y \geq 0$

$250 - 6x \geq 0$

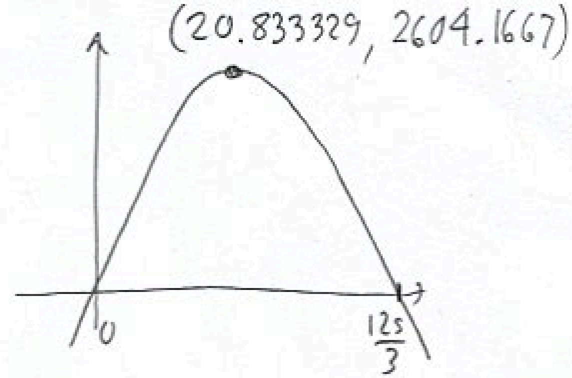
$6x \leq 250$

$x \leq \dfrac{250}{6} = \dfrac{125}{3}$

Domain

$\left[0, \dfrac{125}{3}\right] = \left[0, 41\tfrac{2}{3}\right]$

b)



Maximal area occurs for a block of size

20.833329×125