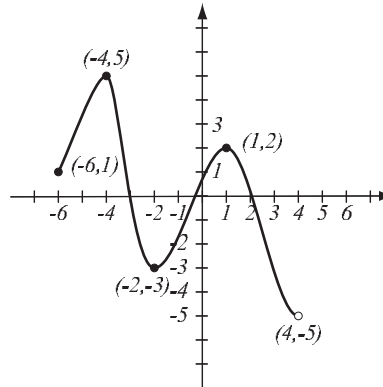


1. (8pts) Use the graph of the function f at right to answer the following questions.

- Find $f(-2)$ and $f(6)$.
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = 3$?



2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3 - 3x^2 - 5x - 6$. Draw the graph here, and indicate units on the axes. Find all the x - and y -intercepts (accuracy: 6 decimal points).

3. (4pts) Convert to scientific notation or a decimal number:

$$3.52 \times 10^{-3} =$$

$$12,743,300 =$$

Use formulas to expand:

4. (4pts) $(4x - 3y)^2 =$

5. (3pts) $(x^2 - 5)(x^2 + 5) =$

6. (5pts) Factor: $x^3 - 64 =$

Simplify, showing intermediate steps. Assume variables can be any real numbers.

7. (2pts) $\sqrt{75} =$

8. (5pts) $\sqrt{32x^5y^2} =$

9. (8pts) Simplify.

$$\frac{3x}{x^2 + 5x - 24} - \frac{x + 1}{x^2 - 64} =$$

10. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to radical notation.

$$\frac{\left(x^{-5}y^{\frac{7}{2}}\right)^{\frac{2}{5}}}{\left(x^{\frac{5}{4}}y^{-\frac{4}{5}}\right)^2} =$$

11. (6pts) Rationalize the denominator.

$$\frac{3\sqrt{5} - 5\sqrt{2}}{\sqrt{2} + \sqrt{5}}$$

12. (5pts) Solve the equation for x .

$$ax + by = 5x$$

13. (8pts) Find the domain of the function $f(x) = \frac{3 + 7\sqrt{x}}{x^2 - 8x + 15}$ and write it using interval notation.

14. (9pts) Let $g(x) = \frac{3x - 7}{x^2 + 5}$. Find the following (simplify where appropriate).

$$g(-2)$$

$$g(0)$$

$$g(\sqrt{z + 1})$$

$$g(x - 3)$$

15. (5pts) Think of equations whose graphs you are familiar with.

a) Write one equation whose graph is not the graph of a function. Why?

b) Write one equation whose graph is the graph of a function. Why?

c) Draw both graphs.

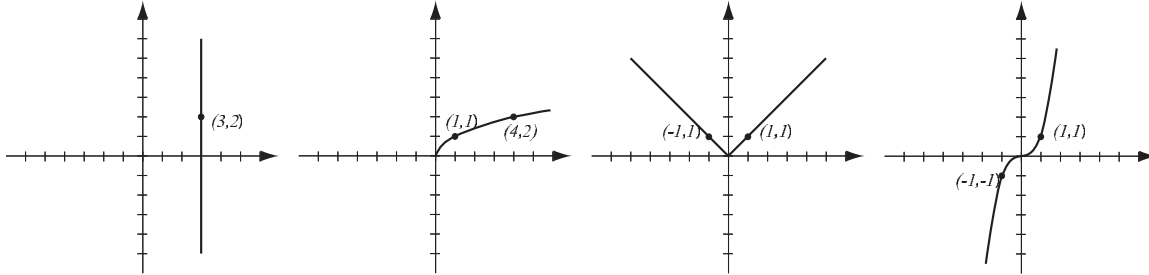
16. (10pts) The diameter of a circle has endpoints $(-2, 3)$ and $(4, -1)$.

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

Bonus (10pts) Let $C = (x, y)$ be any point on the circle of radius r centered at the origin, and let $A = (-r, 0)$ and $B = (r, 0)$ be the endpoints of its diameter. Draw the picture and show that the triangle ABC is a right triangle. (*Hint: use the distance formula and the equation of the circle.*)

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through $(1, 2)$ and $(3, -4)$. Then check if this line is perpendicular to the line $x - 4y = -12$. Draw both lines.

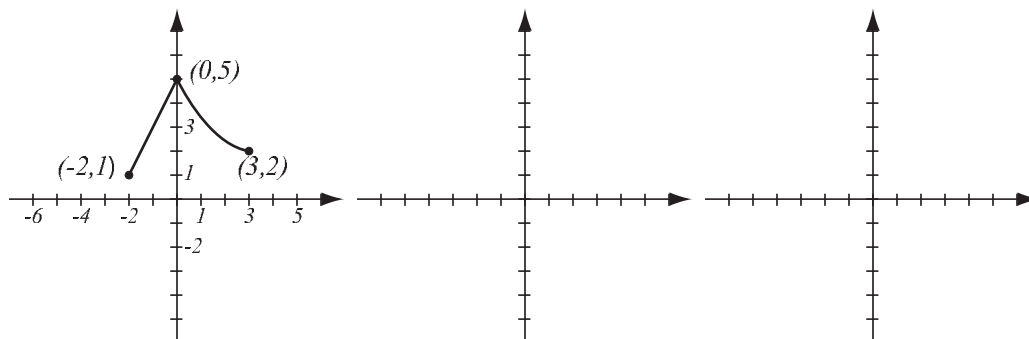
3. (5pts) Solve the inequality and write your solution in interval notation.

$$3x - 1 \leq 9 \text{ or } 2x - 5 > 9$$

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x - 2, & \text{if } -4 < x \leq -1 \\ 5 - 2x, & \text{if } x > -1 \end{cases}$$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x - 3)$ and $f(2x) + 3$ and label all the relevant points.



6. (14pts) Let $f(x) = \frac{5}{\sqrt{3x - 2}}$, $g(x) = 2x - 1$.

Find the following (simplify where possible):

$$(f + g)(7) =$$

$$(fg)(x) =$$

$$(g \circ f)(2) =$$

$$(f \circ g)(x) =$$

The domain of f in interval notation

7. (3pts) Consider the function $h(x) = \frac{3}{x^2 - 2x + 4}$. Find functions f and g , neither of which is the “stupid” one, so that $h(x) = f(g(x))$.

- 8.** (16pts) Let $f(x) = x^3 - 2x^2 - 3x$ (answer with 6 decimal points accuracy).
- Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
 - Determine algebraically whether f is even, odd, or neither. Then state how the graph supports your conclusion.
 - Find the local maxima and minima for this function.
 - State the intervals where the function is increasing and where it is decreasing.

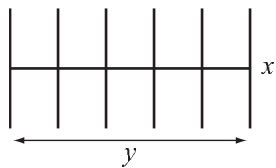
- 9.** (12pts) Marissa has \$300,000 to invest and can split this money between an investment bringing 6% interest, and one bringing 8.5% interest. What is the most she can invest at 6% interest in order to meet a goal of annual interest of at least \$22,000?

10. (14pts) Jack drives from Louisville to Frankfort in 1 hour. On his way back he drives 14mph slower due to traffic, so it takes him 1 hour and 20 minutes.

- a) What is Jack's speed on the way there?
- b) How far is Louisville from Frankfort?

Bonus. (14pts) That same farmer's market wishes to build the same block of 10 stalls separated by walls (see picture) so that the area of the block (total area of stalls) is 2500 square feet. The market wishes to minimize the length of the walls used in the block.

- a) Express the length of the walls used in the block as a function of the length of one of the sides x . What is the domain of this function?
- b) Graph the function in order to find the minimum. What are the dimensions of the block that give the minimum wall length?



College Algebra — Exam 3
MAT 140, Fall 2015 — D. Ivanišić

Name: _____
Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $2 + 4i + 3i(1 - 2i) =$

2. (6pts) $\frac{1 + i}{2 - 3i} =$

3. (4pts) Simplify and justify your answer.

$i^{182} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 10x + 8 = 0$

5. (4pts) Solve the equation.

$|3x - 1| = 7$

6. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 7| \geq 3$

7. (14pts) The quadratic function $f(x) = x^2 - 8x + 19$ is given. Do the following without using the calculator.

- a) Find the x - and y -intercepts of its graph, if any.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.
- d) State the range of f .

Solve the equations:

8. (8pts) $\frac{x}{x+4} + 1 = \frac{x^2 - 6x - 20}{x^2 + 3x - 4}$

9. (8pts) $x + \sqrt{7x + 29} = -3$

10. (14pts) The polynomial $f(x) = (x + 3)^2(x - 4)$ is given.

a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the y -intercept.

c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).

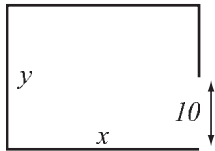
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) In a right triangle, the hypotenuse is 8 inches longer than one of the sides, and 2 inches longer than the other side. What are the lengths of the sides of this triangle?

12. (14pts) A company wishes to build a warehouse with a 10-meter opening on one side (see picture). They have enough money for 400 meters of walls and wish to maximize the area of the warehouse.

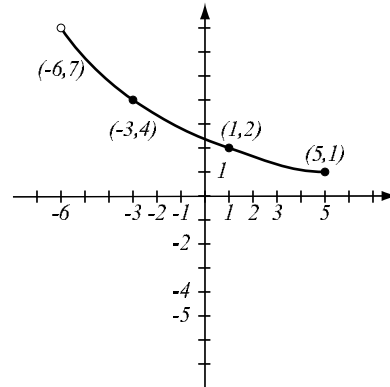
a) Express the area of the warehouse as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible area?



Bonus. (10pts) Find \sqrt{i} , that is, find all complex numbers $x + yi$ so that $(x + yi)^2 = i$. To solve this equation, expand the left side, and solve for x and y using the fact that real and imaginary parts of both sides must be equal.

1. (6pts) The graph of a function f is given.
- Is this function one-to-one? Justify.
 - If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points.



2. (8pts) Let $f(x) = 7x + 5$.
- Find the formula for f^{-1} .
 - Verify that $f^{-1}(f(x)) = x$.
3. (10pts) Let $f(x) = \frac{2x - 7}{3x + 1}$.
- Find the formula for f^{-1} .
 - Find the range of f .

4. (8pts) Evaluate without using the calculator:

$$\log_2 32 =$$

$$\log_8 \frac{1}{64} =$$

$$\log_a \sqrt[3]{a^5} =$$

$$\log_{b^2} b^8 =$$

5. (4pts) Use the change-of-base formula and your calculator to find $\log_3 21$ with accuracy 6 decimal places. Show how you obtained your number.

6. (6pts) Convert the following exponential or logarithmic equations into logarithmic or exponential equations.

$$e^x = 4$$

$$\log_3 7 = y$$

$$a^4 = 9$$

7. (12pts) Using transformations, draw the graphs of $f(x) = -2^x + 1$ and $g(x) = 4 \ln(x - 3)$. Explain how you transform graphs of basic functions in order to get the graphs of f and g . Indicate at least one point on the graph and any asymptotes.

8. (6pts) Find the domain of the function $f(x) = \ln(3x - 1) + \ln(5 - 2x)$ and write it in interval notation.

9. (8pts) How much should you invest in an account bearing 3.27%, compounded monthly, if you wish to have \$5,000 in five years?

Solve the equations.

10. (4pts) $7^{3x} = 41$

11. (6pts) $3^{2x-3} = \left(\frac{1}{9}\right)^{x+1}$

12. (8pts) $4^{x-2} = 7^{x+1}$

13. (14pts) The US population was 249 million in 1990 and 309 million in 2010. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.

a) Find k and write the function that describes the population at time t years since 1990. Graph it on paper.

b) When will population reach 350 million, according to the model?

Bonus (10pts) Solve the equation.

$$\log_2(x + 1) + \log_2(x + 3) = 3$$