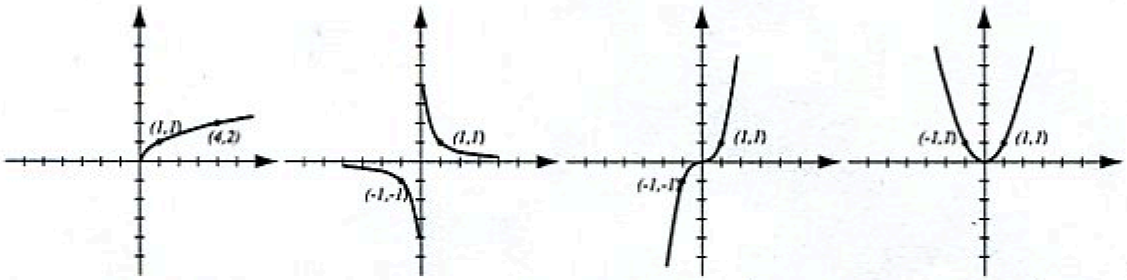
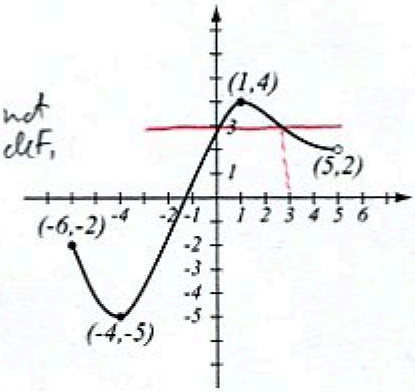
College Algebra — Final Exam	Name: Saul Ocean
MAT 140, Fall 2015 — D. Ivanšić	*Show all your work!*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=\sqrt{x}$ $\qquad$ $y=\frac{1}{x}$ $\qquad$ $y=x^3$ $\qquad$ $y=x^2$

2. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find $f(-4)$ and $f(5)$. $f(-4)=-5$, $f(5)$ not def.

b) What is the domain of f? $[-6,5)$

c) What is the range of f? $[-5,4]$

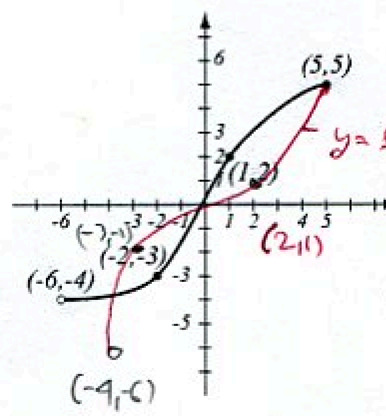
d) What are the solutions of the equation $f(x)=3$?

$x=0,3$

3. (6pts) The graph of a function f is given.

a) Is this function one-to-one? Justify.

b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) yes - it passes the horizontal line test

4. (9pts) Find the equation of the line (in form $y=mx+b$) that passes through $(1,-3)$ and is parallel to the line $x-3y=9$. Draw both lines.

$x-3y=9$

$3y=x-9$

$y=\frac{1}{3}x-3$

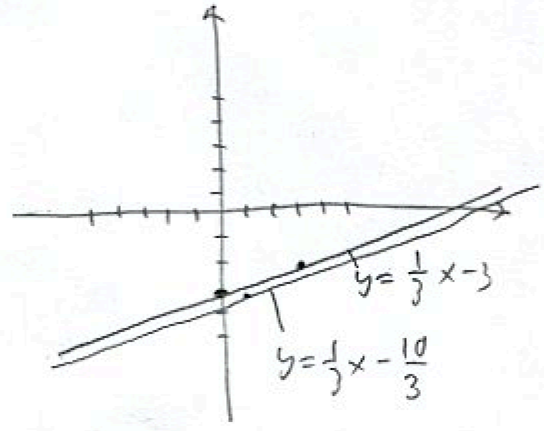
slope $=\frac{1}{3}$

Parallel line has same slope

$y-(-3)=\frac{1}{3}(x-1)$

$y+3=\frac{1}{3}x-\frac{1}{3}$

$y=\frac{1}{3}x-\frac{10}{3}$

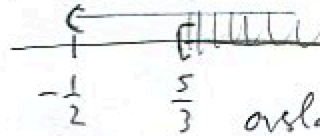


5. (6pts) Find the domain of the function $f(x) = \ln(2x+1) + \sqrt{3x-5}$ and write it in interval notation.

Must have $2x+1 > 0$ and $3x - 5 \geq 0$

$2x > -1$ $\quad$ $3x \geq 5$

$x > -\frac{1}{2}$ and $\quad$ $x \geq \frac{5}{3}$



overlap is

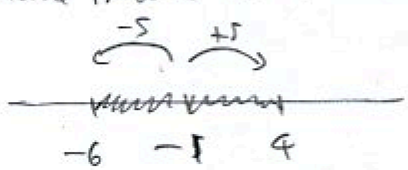
$[\frac{5}{3}, \infty)$

6. (6pts) Solve the inequality. Write the solution in interval form.

$|x+1| \leq 5$

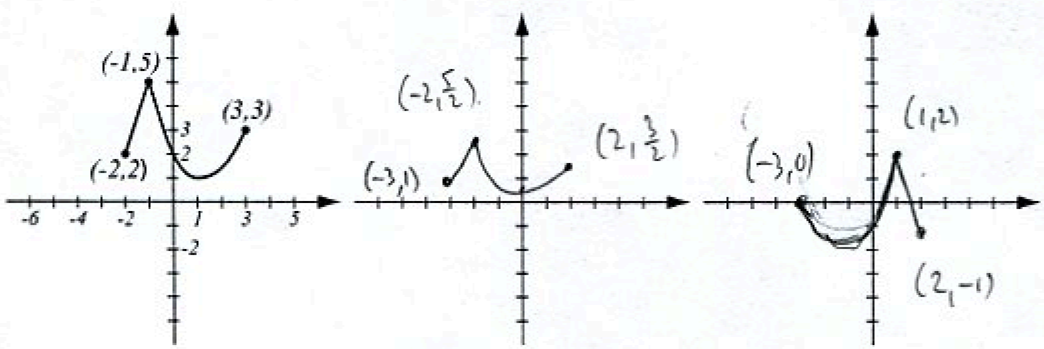
$|x - (-1)| \leq 5$

distance from x to $-1 \leq 5$



$[-6, 4]$

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $\frac{1}{2}f(x+1)$ and $f(-x) - 3$ and label all the relevant points.



shift left 1
stretch vertically,
factor = $\frac{1}{2}$

reflect in y-axis,
shift down 3

8. (12pts) The quadratic function $f(x) = -x^2 - 4x + 5$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

a) y-int. $f(0) = 5$

x-int. $-x^2 - 4x + 5 = 0$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

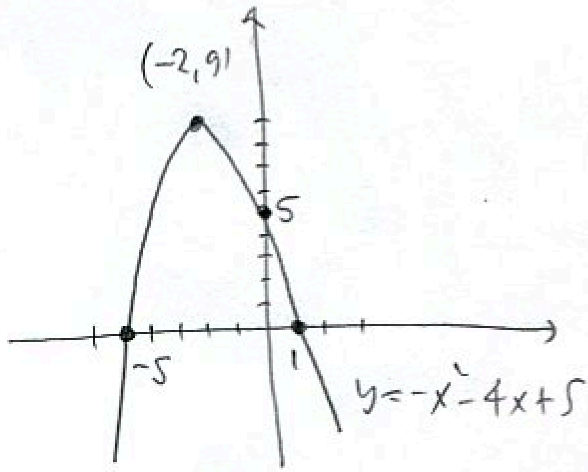
$x = -5, 1$

b) $h = -\frac{b}{2a} = -\frac{-4}{2(-1)} = -2$

$k = f(-2) = -(-2)^2 - 4(-2) + 5$

$= -4 + 8 + 5 = 9$

c)



Simplify and write the answer so all exponents are positive:

9. (7pts) $\frac{(3a^{-3}b^4)^2}{(6a^{-3}b^5)^3} = \frac{3^2 (a^{-3})^2 (b^4)^2}{6^3 (a^{-3})^3 (b^5)^3} = \frac{\cancel{9} a^{-6} b^8}{6 \cdot \cancel{36}\, a^{-9} b^{15}} = \frac{a^{-6-(-9)} b^{8-15}}{24}$

4

$= \frac{a^3 b^{-7}}{24} = \frac{a^3}{24 b^7}$

10. (8pts) Simplify.

$\frac{2x}{x^2 + 2x - 8} - \frac{x+3}{x^2 - 16} = \frac{2x}{(x-2)(x+4)} - \frac{x+3}{(x-4)(x+4)}$

$= \frac{2x(x-4) - (x+3)(x-2)}{(x-2)(x+4)(x-4)}$

	±	±
prod = 6	1, 6	2, 3
sum = -9	no	no

$= \frac{2x^2 - 8x - (x^2 + x - 6)}{(x-2)(x+4)(x-4)}$

$= \frac{x^2 - 9x + 6}{(x-2)(x+4)(x-4)}$ ← doesn't factor

$\frac{x-3}{(x-2)(x+4)}$

11. (14pts) The polynomial $f(x) = x(x-4)(x+3)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

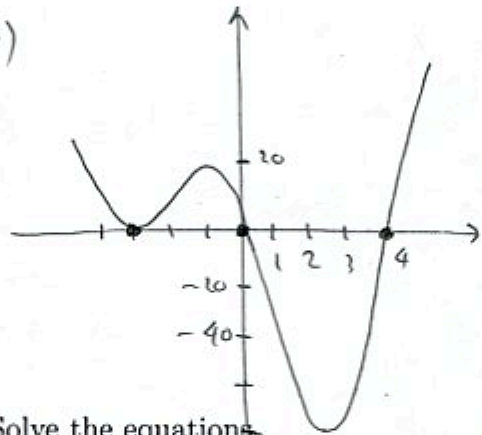
a) like $x \cdot x \cdot x^2 = x^4$ $\cup$

b)

zero	0	4	−3
mult	1	1	2

y-int: $f(0)=0$

c)



d) Turning points:
$(-3,0)$ local min $f(-3)=0$
$(-1.137457, 20.27)$ local max $f(-1.137457)=20.27196$
$(-2.637458, -114.\_)$ local min $f(2.637458)=-114.2095$

Solve the equations.

12. (8pts) $2 + \sqrt{46-3x} = x$

$\sqrt{46-3x} = x-2 \quad |^2$

$46-3x = x^2-4x+4$

$x^2 - x - 42 = 0$

$(x+6)(x-7)=0$

$x = -6, \boxed{7}$

check:

$2+\sqrt{46-3\cdot(-6)} \overset{?}{=} -6$

$2+\sqrt{64} \overset{?}{=} -6$ no

$2+\sqrt{46-3\cdot 7} = 7$

$2+\sqrt{25} = 7$ yes

13. (8pts) $e^{x-3} = 5^{x+7}$ $|\ln$

$\ln e^{x-3} = \ln 5^{x+7}$

$x-3 = (x+7)\ln 5$

$x-3 = x\ln 5 + 7\ln 5$

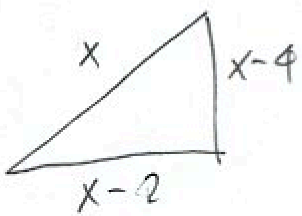
$x - x\ln 5 = 3+7\ln 5$

$x(1-\ln 5) = 3+7\ln 5$

$x = \dfrac{3+7\ln 5}{1-\ln 5}$

$= -23.408562$

14. (12pts) In a right triangle, one side of the triangle is 4 inches shorter than the hypothenuse, and the other side is 2 inches shorter than the hypothenuse. What are the lengths of the sides of this triangle?



$(x-2)^2+(x-4)^2=x^2$

$x^2-4x+4+x^2-8x+16=x^2$

$x^2-12x+20=0$

$(x-10)(x-2)=0$

sides: 10, 6, 8

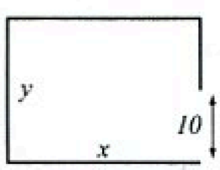
$x=10, 2$

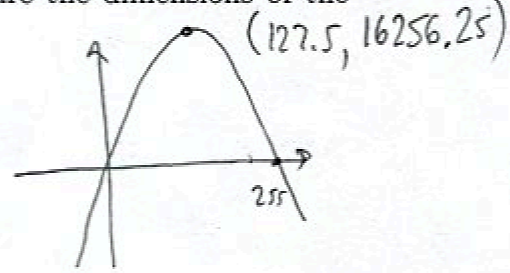
doesn't fit context since $x\geq 4$
(has to be 4 greater than a side)

15. (14pts) A company wishes to build a warehouse with a 10-meter opening on one side (see picture). They have enough money for 500 meters of walls and wish to maximize the area of the warehouse.

a) Express the area of the warehouse as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible area?





a) $A=xy$

$500=2x+y+y-10$

$500=2x+2y-10$

$510=2x+2y$

$y=\frac{510-2x}{2}=255-x$

$A=x(255-x)=-x^2+255x$

Domain:
must have
$x\geq 0$
$y\geq 10$ (for opening)

$255-x\geq 10$
$245\geq x$

$[0,245]$

$h=-\frac{255}{2\cdot(-1)}=127.5$

$x=127.5$

$y=255-127.5=127.5$

Greatest area for
127.5×127.5 rectangle

16. (14pts) The US population was 249 million in 1990 and 309 million in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 1990. Graph it on paper.
b) When will population reach 350 million, according to the model?

a) $P(t) = 249 e^{kt}$

Need k

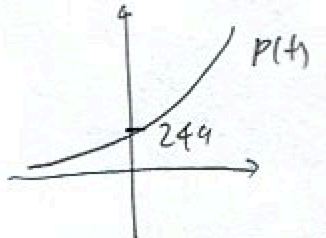
$309 = P(20) = 249 e^{k \cdot 20}$

$\frac{309}{249} = e^{k \cdot 20} \quad | \ln$

$\ln \frac{309}{249} = k \cdot 20$

$k = \frac{\ln \frac{309}{249}}{20} = 0.0107944$

$P(t) = 249 e^{0.0107944t}$



b) $249 e^{0.0107944t} = 350$

$e^{0.0107\ldots t} = \frac{350}{249} \quad | \ln$

$0.0107\ldots t = \ln \frac{350}{249}$

$t = \frac{\ln \frac{350}{249}}{0.0107944} = 31.54225$

About 32 years, so, in 2022.

Bonus. (10pts) Find $\sqrt{i}$, that is, find all complex numbers $x + yi$ so that $(x + yi)^2 = i$. To solve this equation, expand the left side, and solve for x and y using the fact that real and imaginary parts of both sides must be equal.

$(x + yi)^2 = i$

$x^2 + 2xyi + y^2 i^2 = i$

$x^2 - y^2 + 2xyi = i$

Comparing real and imaginary parts, we get:

$x^2 - y^2 = 0$ and $2xy = 1$

$y = \frac{1}{2x}$

$x^2 - \left(\frac{1}{2x}\right)^2 = 0$

$x^2 - \frac{1}{4x^2} = 0 \quad \cdot 4x^2$

$4x^4 - 1 = 0$

$x^4 = \frac{1}{4}$

$x^2 = \frac{1}{2}$

$\frac{2}{\sqrt{2}} = \frac{\sqrt{2} \cdot \sqrt{2}}{\sqrt{2}} = \sqrt{2}$

$x = \pm \frac{1}{\sqrt{2}}, \; y = \frac{1}{2\left(\pm \frac{1}{\sqrt{2}}\right)} = \frac{1}{\pm \sqrt{2}}$

Solutions are

$\pm \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)$