College Algebra — Exam 4
MAT 140, Fall 2015 — D. Ivanšić

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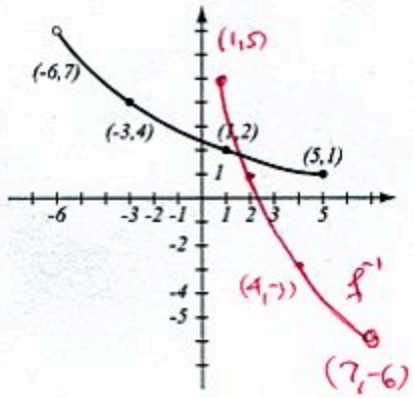
Show all your work!

1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) yes. it passes the vertical line test

2. (8pts) Let $f(x) = 7x + 5$.
a) Find the formula for f^{-1}.
b) Verify that $f^{-1}(f(x)) = x$.

a) $y = 7x+5$

$y-5 = 7x$

$x = \frac{y-5}{7}$

$f^{-1}(y) = \frac{y-5}{7}$

b) $f^{-1}(f(x)) = f^{-1}(7x+5) = \frac{7x+5-5}{7} = \frac{7x}{7} = x$

3. (10pts) Let $f(x) = \frac{2x-7}{3x+1}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

a) $y = \frac{2x-7}{3x+1}$

$y(3x+1) = 2x-7$

$3xy + y = 2x-7$

$3xy - 2x = -y-7$

$x(3y-2) = -y-7$

$x = \frac{-y-7}{3y-2}$

$f^{-1}(y) = \frac{-y-7}{3y-2}$

b) range f = domain f^{-1}

$3y-2 = 0$

$3y = 2$

$y = \frac{2}{3}$

Range $f = \{y \mid y \neq \frac{2}{3}\}$

$= (-\infty, \frac{2}{3}) \cup (\frac{2}{3}, \infty)$

4. (8pts) Evaluate without using the calculator:

$\log_2 32 = 5$ $\quad$ $\log_8 \frac{1}{64} = -2$ $\quad$ $\log_a \sqrt[3]{a^5} = \frac{5}{3}$ $\quad$ $\log_{b^2} b^8 = 4$

$2^? = 32$ $\quad$ $8^? = \frac{1}{64} = \frac{1}{8^2} = 8^{-2}$ $\quad$ $a^? = \sqrt[3]{a^5} = a^{\frac{5}{3}}$ $\quad$ $(b^2)^? = b^8$

5. (4pts) Use the change-of-base formula and your calculator to find $\log_3 21$ with accuracy 6 decimal places. Show how you obtained your number.

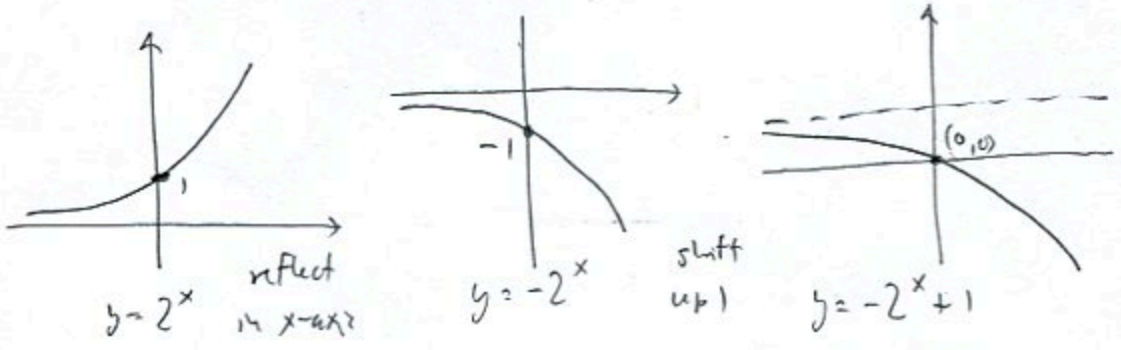
$\log_3 21 = \frac{\ln 21}{\ln 3} = 2.771244$

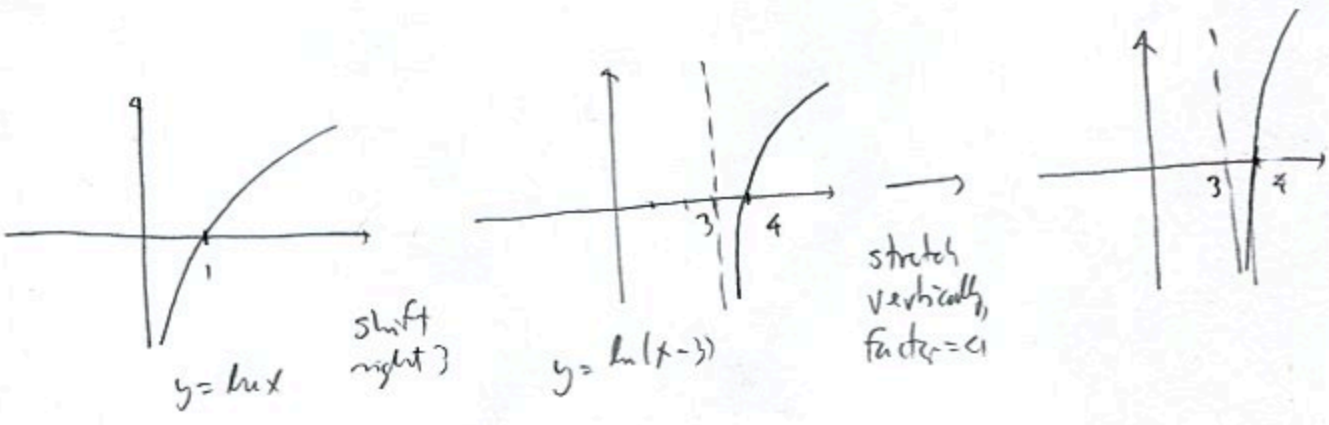
6. (6pts) Convert the following exponential or logarithmic equations into logarithmic or exponential equations.

$e^x = 4$ $\quad$ $\log_3 7 = y$ $\quad$ $a^4 = 9$

$x = \ln 4$ $\quad$ $3^y = 7$ $\quad$ $\log_a 9 = 4$

7. (12pts) Using transformations, draw the graphs of $f(x) = -2^x + 1$ and $g(x) = 4\ln(x-3)$. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least one point on the graph and any asymptotes.





8. (6pts) Find the domain of the function $f(x) = \ln(3x - 1) + \ln(5 - 2x)$ and write it in interval notation.

Must have

$3x-1>0$ and $5-2x>0$

$3x>1$ $\quad 5>2x$

$x>\frac{1}{3}$ and $\frac{5}{2}>x$

$\left(\frac{1}{3}, \frac{5}{2}\right)$

9. (8pts) How much should you invest in an account bearing 3.27%, compounded monthly, if you wish to have \$5,000 in five years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$5000 = P\left(1+\frac{0.0327}{12}\right)^{12\cdot 5}$

$5000 = P\cdot(1.002725)^{60}$

$5000 = P\cdot 1.177\ldots$

$P = \frac{5000}{1.177\ldots} = 4246.78$

Solve the equations.

10. (4pts) $7^{3x} = 41$ | apply log

$\log 7^{3x} = \log 41$

$3x \log 7 = \log 41$

$x = \frac{\log 41}{3\log 7} = 0.636133$

11. (6pts) $3^{2x-3} = \left(\frac{1}{9}\right)^{x+1}$

$3^{2x-3} = (3^{-2})^{x+1}$

$3^{2x-3} = 3^{-2x-2}$

$2x-3 = -2x-2$

$4x = 1$

$x = \frac{1}{4}$

12. (8pts) $4^{x-2} = 7^{x+1}$ | apply ln

$\ln 4^{x-2} = \ln 7^{x+1}$

$(x-2)\ln 4 = (x+1)\ln 7$

$x\ln 4 - 2\ln 4 = x\ln 7 + \ln 7$

$x\ln 4 - x\ln 7 = \ln 7 + 2\ln 4$

$x(\ln 4 - \ln 7) = \ln 7 + 2\ln 4$

$x = \frac{\ln 7 + 2\ln 4}{\ln 4 - \ln 7}$

$= -8.431676$

13. (14pts) The US population was 249 million in 1990 and 309 million in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 1990. Graph it on paper.
b) When will population reach 350 million, according to the model?

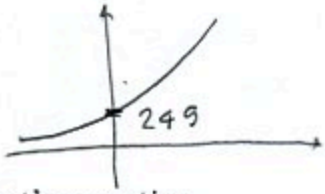
a) $P(t) = 249e^{kt}$

$309 = 249e^{k \cdot 20}$

$\frac{309}{249} = e^{k \cdot 20} \quad | \ln$

$\ln \frac{309}{249} = k \cdot 20$

$k = \frac{\ln \frac{309}{249}}{20} = 0.0107944$



b) $350 = 249e^{0.0107944t}$

$\frac{350}{249} = e^{0.01 \cdot t}$

$\ln \frac{350}{249} = 0.0107 \cdot t$

$t = \frac{\ln \frac{350}{249}}{0.0107944} = 31.54225$ years

About 32 years from 1990, that is, in 2022

Bonus (10pts) Solve the equation.

$\log_2(x+1) + \log_2(x+3) = 3$

$\log_2((x+1)(x+3)) = 3$

$(x+1)(x+3) = 2^3$

$x^2 + 4x + 3 = 8$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x = -5, 1$

Checks:

$x = -5$ $\log_2(-4) + \log_2(-2) \stackrel{?}{=} 3$

$\log_2$(neg.) not defined

$x = 1$ $\log_2 2 + \log_2 4 \stackrel{?}{=} 3$

$1 + 2 = 3$ yes

Solution: $x = 1$