College Algebra — Exam 3
MAT 140, Fall 2015 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $2 + 4i + 3i(1 - 2i) =$ $2+4i+3i-6i^2$

$= 8+7i$

2. (6pts) $\dfrac{1+i}{2-3i} = \dfrac{1+i}{2-3i}\cdot\dfrac{2+3i}{2+3i} = \dfrac{2+3i+2i+3i^2}{2^2-(3i)^2} = \dfrac{2+5i-3}{4-(-9)} = \dfrac{-1+5i}{13} = -\dfrac{1}{13}+\dfrac{5}{13}i$

3. (4pts) Simplify and justify your answer.

$i^{182} = \underbrace{i^{180}}\cdot i^2 = 1\cdot i^2 = -1$

$= (i^4)^{45} = 1^{45} = 1$

4. (6pts) Solve the equation by completing the square.

$x^2 - 10x + 8 = 0 \quad | +25$

$x^2 - 10x + 25 + 8 = 25$

$x^2 - 2\cdot x\cdot 5 + 5^2 = 17$

$(x-5)^2 = 17$

$x - 5 = \pm\sqrt{17}$

$x = 5 \pm \sqrt{17}$

5. (4pts) Solve the equation.

$|3x - 1| = 7$

$3x - 1 = 7$ or $3x - 1 = -7$

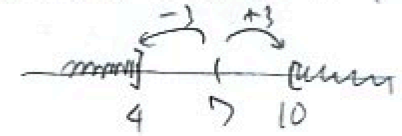
$3x = 8 \qquad 3x = -6$

$x = \dfrac{8}{3}$ or $x = -2$

6. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 7| \geq 3$

distance from x to $7 \geq 3$



$(-\infty, 4] \cup [10, \infty)$

7. (14pts) The quadratic function $f(x) = x^2 - 8x + 19$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
d) State the range of f.

b) $h = -\frac{b}{2a} = -\frac{-8}{2\cdot 1} = 4$

$k = f(4) = 4^2 - 8\cdot 4 + 19 = 3$

a) y-int: $f(0) = 19$

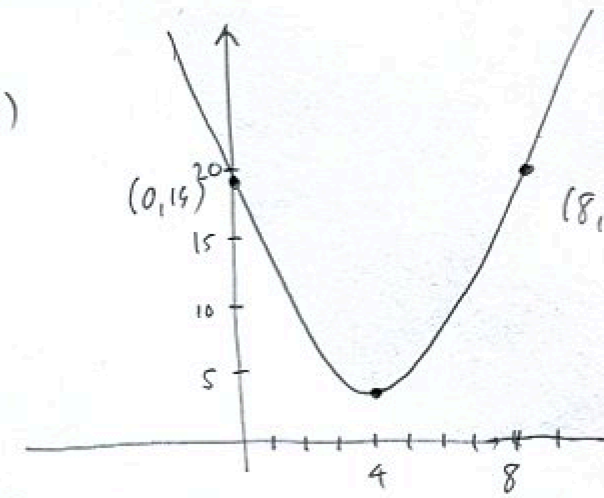
x-int: $x^2 - 8x + 19 = 0$

$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4\cdot 1\cdot 19}}{2\cdot 1}$

$= \frac{8 \pm \sqrt{64-76}}{2} = \frac{8 \pm \sqrt{-12}}{2}$

no real solutions

c)



d) Range $= [3, \infty)$

Solve the equations:

8. (8pts) $\frac{x}{x+4} + 1 = \frac{x^2 - 6x - 20}{x^2 + 3x - 4}$ $\Big|\cdot (x+4)(x-1)$

$(x+4)(x-1)$

$\frac{x}{\cancel{x+4}}\cancel{(x+4)}(x-1) + (x+4)(x-1) = \frac{x^2-6x-20}{\cancel{x^2+3x-4}}\cancel{(x+4)(x-1)}$

$x(x-1) + x^2+3x-4 = x^2-6x-20$

$x^2 - x + x^2 + 3x - 4 = x^2 - 6x - 20$

$2x^2 + 2x - 4 = x^2 - 6x - 20$

$x^2 + 8x + 16 = 0$

$(x+4)^2 = 0$

$x = -4$ – gives 0 in denom

so no solution

9. (8pts) $x + \sqrt{7x + 29} = -3$

$\sqrt{7x+29} = -3-x$ $\Big|^2$

$(-3-x)^2 = (-(x+3))^2 = (x+3)^2$

$7x + 29 = x^2 + 6x + 9$

$x^2 - x - 20 = 0$

$(x-5)(x+4) = 0$

$x = 5, -4$

check: $5 + \sqrt{35+29} \overset{?}{=} -3$

$5 + 8 \overset{?}{=} -3$ no

$-4 + \sqrt{-28+29} = -3$

$-4 + 1 = -3$ yes

Only $x = -4$ is the solution

10. (14pts) The polynomial $f(x) = (x+3)^2(x-4)$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

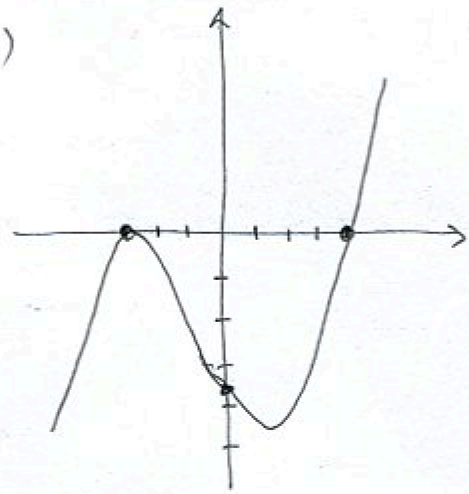
a) like $x^2 \cdot x = x^3$

b)

zero	-3	4
mult	2	1

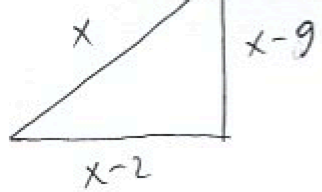
y-int: $f(0) = (0+3)^2(-4)$
$= -36$

c)



d) local max $f(-3) = 0$
local min $f(1.666667) = -50.81481$

11. (12pts) In a right triangle, the hypothenuse is ~~8~~ 9 inches longer than one of the sides, and 2 inches longer than the other side. What are the lengths of the sides of this triangle?



x = length of hypothenuse

Pythagorean Theorem:

$(x-2)^2 + (x-9)^2 = x^2$

$x^2 - 4x + 4 + x^2 - 18x + 81 = x^2$

$2x^2 - 20x + 68 = x^2$

$x^2 - 22x + 85 = 0$

$(x-5)(x-17) = 0$

$x = 5, 17$

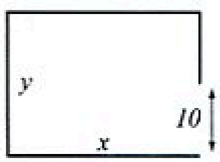
$\uparrow$ doesn't fit context, since $x \geq 9$

Side-lengths are
15, 8, 17

12. (14pts) A company wishes to build a warehouse with a 10-meter opening on one side (see picture). They have enough money for 400 meters of walls and wish to maximize the area of the warehouse.

a) Express the area of the warehouse as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible area?



a) $2x+y+y-10=400$

$2x+2y=410$

$x+y=205$

$y=205-x$

$A=xy=x(205-x)=-x^2+205x$

Must have $x \geq 0$

$y \geq 10$

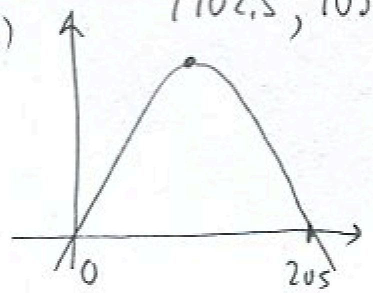
$205-x \geq 10$

$x \leq 195$

$0 \leq x \leq 195$, $[0,195]$

b)

$(102.5,\ 10506.25)$



Vertex is at $x=-\frac{b}{2a}=-\frac{205}{2\cdot(-1)}=102.5$

Biggest warehouse is 102.5×102.5

with area 10506.25

Bonus. (10pts) Find $\sqrt{i}$, that is, find all complex numbers $x+yi$ so that $(x+yi)^2=i$. To solve this equation, expand the left side, and solve for x and y using the fact that real and imaginary parts of both sides must be equal.

$(x+yi)^2=i$

$x^2+2xyi+(yi)^2=i$

$x^2-y^2+2xyi=0+1i$

So

$x^2-y^2=0 \Rightarrow x^2=y^2$,

$2xy=1 \Rightarrow y=\frac{1}{2x}$

$\Rightarrow x^2=\left(\frac{1}{2x}\right)^2$

$x^2=\frac{1}{4x^2}$

$x^4=\frac{1}{4}$ $\sqrt{\ }$

$x^2=\frac{1}{2}$

$x=\pm\frac{1}{\sqrt{2}}$ $y=\frac{1}{2\left(\pm\frac{1}{\sqrt{2}}\right)}=\pm\frac{\sqrt{2}}{2}=\pm\frac{1}{\sqrt{2}}$

Solutions: $\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}i$, $-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}i$