College Algebra — Exam 2 MAT 140, Fall 2015 — D. Ivanšić

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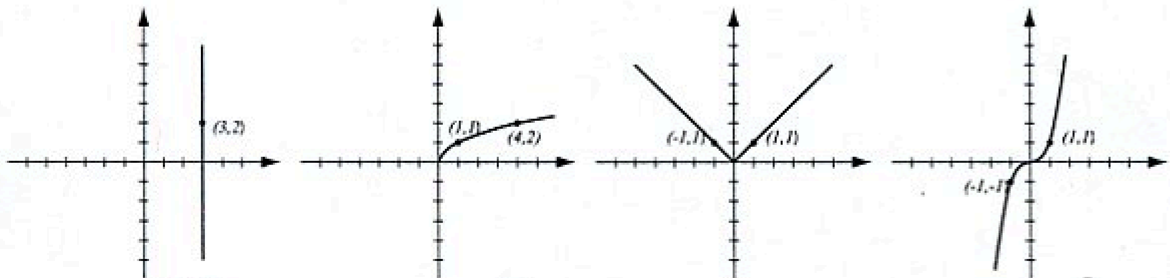
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$x=3$ $\qquad$ $y=\sqrt{x}$ $\qquad$ $y=|x|$ $\qquad$ $y=x^3$

2. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through $(1, 2)$ and $(3, -4)$. Then check if this line is perpendicular to the line $x - 4y = -12$. Draw both lines.

$m = \frac{-4-2}{3-1} = \frac{-6}{2} = -3$

$y-2 = -3(x-1)$

$y-2 = -3x+3$

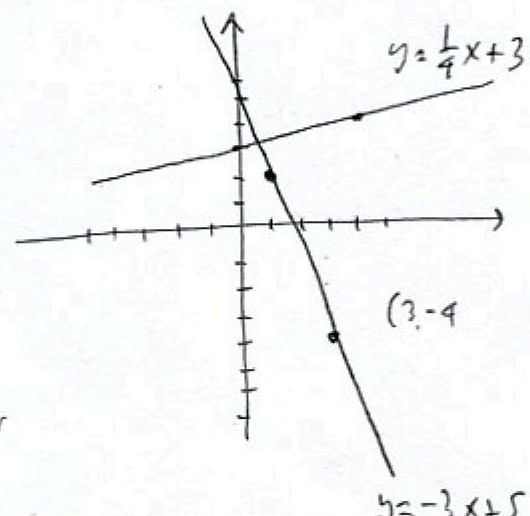
$y = -3x+5$

$x-4y=-12$

$-4y = -x-12 \quad | \div(-4)$

$y = \frac{1}{4}x+3$

$-\frac{1}{-3} = \frac{1}{3}$, not equal to $\frac{1}{4}$, so not perpendicular

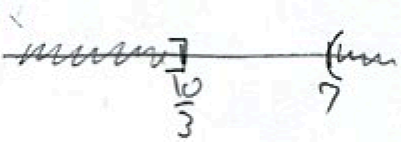


3. (5pts) Solve the inequality and write your solution in interval notation.

$3x - 1 \le 9$ or $2x - 5 > 9$

$3x \le 10 \qquad 2x > 14$

$x \le \frac{10}{3}$ or $x > 7$



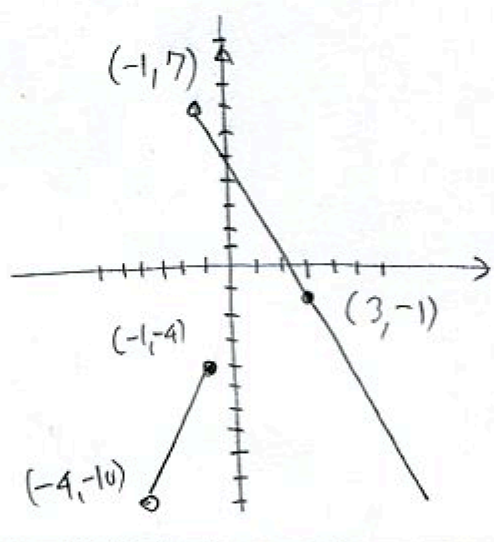
$(-\infty, \frac{10}{3}] \cup (7, \infty)$

4. (8pts) Sketch the graph of the piecewise-defined function:

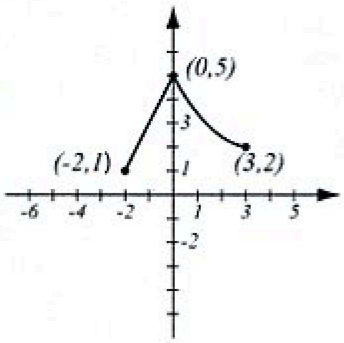
$$f(x) = \begin{cases} 2x - 2, & \text{if } -4 < x \le -1 \\ 5 - 2x, & \text{if } x > -1 \end{cases}$$

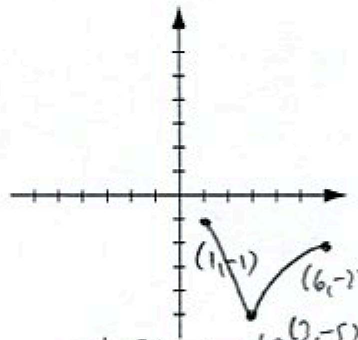
x	$2x-2$
-4	-10
-1	-4

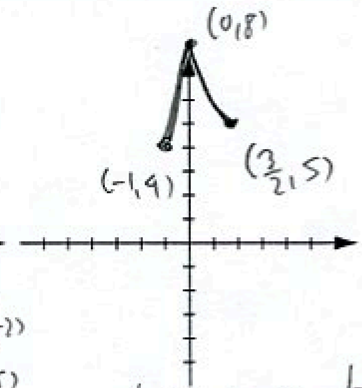
x	$5-2x$
-1	7
3	-1



5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x-3)$ and $f(2x)+3$ and label all the relevant points.







shift right 3
reflect in x-axis

stretch horizontally factor $= \frac{1}{2}$
shift up 3

6. (14pts) Let $f(x) = \dfrac{5}{\sqrt{3x-2}}$, $g(x) = 2x-1$.

Find the following (simplify where possible):

$(f+g)(7) = f(7)+g(7)$

$= \dfrac{5}{\sqrt{21-2}} + 2\cdot 7 - 1 = \dfrac{5}{\sqrt{19}} + 13$

$(fg)(x) = f(x)g(x)$

$= \dfrac{5}{\sqrt{3x-2}}\cdot(2x-1) = \dfrac{5(2x-1)}{\sqrt{3x-2}}$

$(g\circ f)(2) = g(f(2)) = g\left(\dfrac{5}{\sqrt{6-2}}\right)$

$= g\left(\frac{5}{2}\right) = 2\cdot\frac{5}{2} - 1 = 4$

$(f\circ g)(x) = f(g(x)) = f(2x-1)$

$= \dfrac{5}{\sqrt{3(2x-1)-2}} = \dfrac{5}{\sqrt{6x-3-2}} = \dfrac{5}{\sqrt{6x-5}}$

The domain of f in interval notation

Must have $3x-2>0$ (no 0 in denom.)

$3x>2$

$x>\frac{2}{3}$

$\frac{2}{3}$

$\left(\frac{2}{3},\infty\right)$

7. (3pts) Consider the function $h(x) = \dfrac{3}{x^2-2x+4}$. Find functions f and g, neither of which is the "stupid" one, so that $h(x) = f(g(x))$.

$g(x) = x^2 - 2x + 4$

$f(x) = \dfrac{3}{x}$

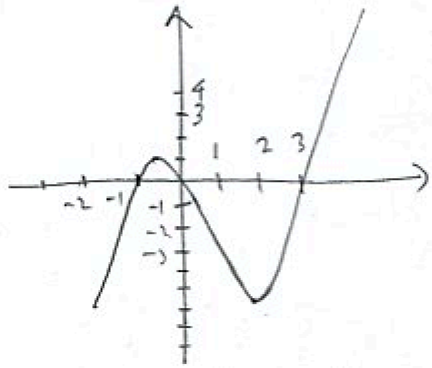
or

$g(x) = x^2 - 2x$

$f(x) = \dfrac{3}{x+4}$

8. (16pts) Let $f(x) = x^3 - 2x^2 - 3x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Then state how the graph supports your conclusion.
c) Find the local maxima and minima for this function.
d) State the intervals where the function is increasing and where it is decreasing.

a)



c) $f(-0.535183)=0.87942$ is a local max

$f(1.868518)=-6.064605$ is a local min

b) $f(-x)=(-x)^3-2(-x)^2-3(-x)$

$=-x^3-2x^2+3x$

$\neq f(x)$ nor $-f(x)$

neither

Graph has no symmetry (bottom loop is bigger)

d) Increasing on

$(-\infty,-0.535183)\cup(1.868518,\infty)$

Decreasing on

$(-0.535183,\ 1.868518)$

9. (12pts) Marissa has \$300,000 to invest and can split this money between an investment bringing 6% interest, and one bringing 8.5% interest. What is the most she can invest at 6% interest in order to meet a goal of annual interest of at least \$22,000?

x= amt invested @ 6%

300000 − x = amt invested at 8.5%

(interest from account @ 6%) + (interest from acct @ 8.5%) ≥ 22,000

$0.06x + (300000-x)0.085 \geq 22{,}000$

$0.06x + 25{,}500 - 0.085x \geq 22{,}000 \quad | -25{,}500$

$-0.025x \geq -3{,}500 \quad | \div -0.025$

$x \leq \frac{-3.500}{-0.025} = 140{,}000$

Can invest at most 140000 into account @ 6%

10. (14pts) Jack drives from Louisville to Frankfort in 1 hour. On his way back he drives 14mph slower due to traffic, so it takes him 1 hour and 20 minutes.
a) What is Jack's speed on the way there?
b) How far is Louisville from Frankfort?

$d, r, 1$ hour

$d, r-14, 1\frac{1}{3}$ hours

1 hr 20 min $= 1 + \frac{1}{3}$ hr

r = Jack's speed on way to Frankfort

same $\left\langle \begin{array}{l} d = r \cdot 1 \\ d = (r-14)(1+\frac{1}{3}) \end{array} \right.$

$r = (r-14)\frac{4}{3}$ $\quad 1 \cdot 3$

$3r = 4(r-14)$

$3r = 4r - 56$

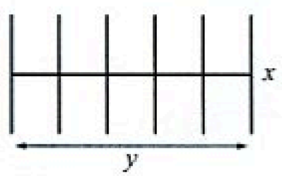
$-r = -56$

$r = 56$

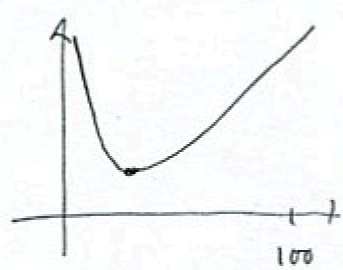
a) speed was 56 mph

b) distance is 56 miles

Bonus. (14pts) That same farmer's market wishes to build the same block of 10 stalls separated by walls (see picture) so that the area of the block (total area of stalls) is 2500 square feet. The market wishes to minimize the length of the walls used in the block.
a) Express the length of the walls used in the block as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the block that give the minimum wall length?



b)



a) $A = xy = 2500$, so $y = \frac{2500}{x}$

$l = 6x + y = 6x + \frac{2500}{x}$

$l(x) = 6x + \frac{2500}{x}$

Domain: must have $x > 0$

$(0, \infty)$

has minimum $l(20.412413) = 244.94897$

Dimensions are $\underset{x}{20.412413} \times \underset{y}{122.474495}$