College Algebra — Exam 1 MAT 140, Fall 2015 — D. Ivanšić

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Show all your work!

1. (8pts) Use the graph of the function f at right to answer the following questions.

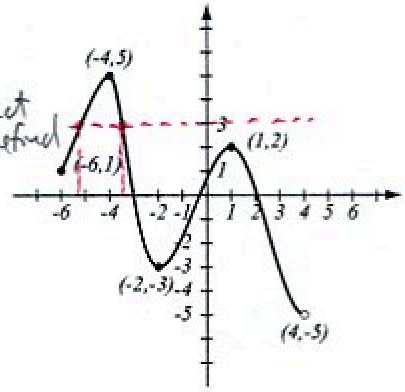
a) Find $f(-2)$ and $f(6)$. $f(-2)=-3$, $f(6)$ not defined

b) What is the domain of f? $[-6,4)$

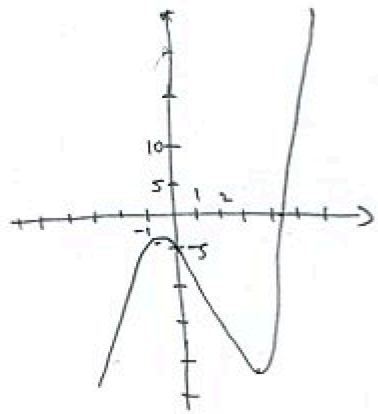
c) What is the range of f? $[-5,5]$

d) What are the solutions of the equation $f(x)=3$?

$x=-5.3, -3.5$



2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3-3x^2-5x-6$. Draw the graph here, and indicate units on the axes. Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).



y int:
$x=0, y=-6$
x-int:
$x=4.433161$

3. (4pts) Convert to scientific notation or a decimal number:

$3.52\times 10^{-3} =$ 0.00352

$12{,}743{,}300 =$ 1.27433×10^{7}

Use formulas to expand:

4. (4pts) $(4x-3y)^2 =$ $(4x)^2 - 2\cdot 4x\cdot 3y + (3y)^2 = 16x^2 - 24xy + 9y^2$

5. (3pts) $(x^2-5)(x^2+5) =$ $(x^2)^2 - 5^2 = x^4 - 25$

6. (5pts) Factor: $x^3-64 =$ $x^3 - 4^3 = (x-4)(x^2+4x+16)$

Simplify, showing intermediate steps. Assume variables can be any real numbers.

7. (2pts) $\sqrt{75} = \sqrt{25 \cdot 3}$

$= 5\sqrt{3}$

8. (5pts) $\sqrt{32x^5y^2} = \sqrt{16 \cdot 2 \cdot x^4 \cdot x \cdot y^2}$

$= 4\sqrt{(x^2)^2}\sqrt{y^2}\sqrt{2x}$

$= 4|x^2||y|\sqrt{2x} = 4x^2|y|\sqrt{2x}$

9. (8pts) Simplify.

$$\frac{3x}{x^2+5x-24} - \frac{x+1}{x^2-64} = \frac{3x(x-8)-(x+1)(x-3)}{(x+8)(x-3)(x-8)}$$

$(x+8)(x-3) \quad (x-8)(x+8)$

$$= \frac{3x^2-24x-(x^2-2x-3)}{(x+8)(x-3)(x-8)} = \frac{2x^2-22x+3}{(x+8)(x-3)(x-8)}$$

$\left.\begin{matrix}\text{prod}=6 \\ \text{sum}=-22\end{matrix}\right\}$ no sol., so $2x^2-22x+3$ doesn't factor

10. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to radical notation.

$$\frac{\left(x^{-5}y^{\frac{7}{2}}\right)^{\frac{2}{5}}}{\left(x^{\frac{5}{4}}y^{-\frac{4}{5}}\right)^2} = \frac{(x^{-5})^{\frac{2}{5}}(y^{\frac{7}{2}})^{\frac{2}{5}}}{(x^{\frac{5}{4}})^2(y^{-\frac{4}{5}})^2} = \frac{x^{-2}y^{\frac{7}{5}}}{x^{\frac{5}{2}}y^{-\frac{8}{5}}} = x^{-2-\frac{5}{2}}y^{\frac{7}{5}-(-\frac{8}{5})} = x^{-\frac{9}{2}}y^{\frac{15}{5}}$$

$$= \frac{y^3}{x^{\frac{9}{2}}} = \frac{y^3}{\sqrt{x^9}}$$

11. (6pts) Rationalize the denominator.

$$\frac{3\sqrt{5}-5\sqrt{2}}{\sqrt{2}+\sqrt{5}} \cdot \frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}} = \frac{3\sqrt{10}-3\sqrt{5}^2-5\sqrt{2}^2+5\sqrt{10}}{\sqrt{2}^2-\sqrt{5}^2} = \frac{8\sqrt{10}-15-10}{2-5}$$

$$= \frac{-25+8\sqrt{10}}{-3} = \frac{25-8\sqrt{10}}{3}$$

12. (5pts) Solve the equation for x.

$ax + by = 5x \quad \Big|\, -5x - by$

$ax - 5x = -by$

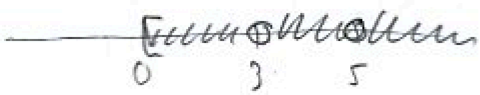
$(a-5)x = -by$

$x = \dfrac{-by}{a-5} = \dfrac{by}{5-a}$

13. (8pts) Find the domain of the function $f(x) = \dfrac{3+7\sqrt{x}}{x^2-8x+15}$ and write it using interval notation.

Must have	Can't have
$x \geq 0$	$x^2 - 8x + 15 = 0$
(due to $\sqrt{x}$)	$(x-5)(x-3) = 0$
	$x = 3, 5$

$\{x \mid x \geq 0,\ x \neq 3, 5\}$



$[0,3) \cup (3,5) \cup (5,\infty)$

14. (9pts) Let $g(x) = \dfrac{3x-7}{x^2+5}$. Find the following (simplify where appropriate).

$g(-2) = \dfrac{3\cdot(-2)-7}{(-2)^2+5} = \dfrac{-13}{9}$

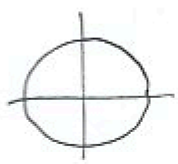
$g(0) = \dfrac{3\cdot 0-7}{0^2+5} = -\dfrac{7}{5}$

$g(\sqrt{z+1}) = \dfrac{3\sqrt{z+1}-7}{(\sqrt{z+1})^2+5} = \dfrac{3\sqrt{z+1}-7}{z+1+5} = \dfrac{3\sqrt{z+1}-7}{z+6}$

$g(x-3) = \dfrac{3(x-3)-7}{(x-3)^2+5} = \dfrac{3x-9-7}{x^2-6x+9+5} = \dfrac{3x-16}{x^2-6x+14}$

15. (5pts) Think of equations whose graphs you are familiar with.
a) Write one equation whose graph is not the graph of a function. Why?
b) Write one equation whose graph is the graph of a function. Why?
c) Draw both graphs.

a) $x^2+y^2=1$ b) $y=x$



Doesn't pass Passes

the horizontal line test

16. (10pts) The diameter of a circle has endpoints $(-2,3)$ and $(4,-1)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

a) center = midpoint

$$=\left(\frac{-2+4}{2}, \frac{3-1}{2}\right)$$

$$=(1,1)$$

Equation:

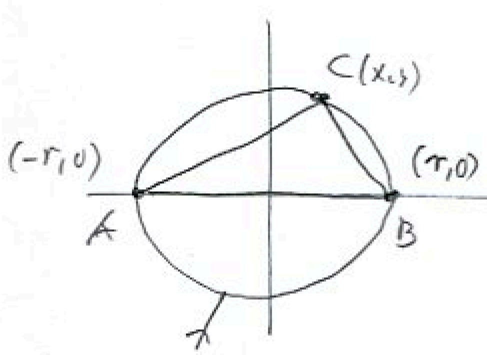
$$(x-1)^2+(y-1)^2=\sqrt{13}^2$$

$$(x-1)^2+(y-1)^2=13$$

radius = $\dfrac{\text{diameter}}{2}$

$$=\frac{\sqrt{(4-(-2))^2+(-1-3)^2}}{2}=\frac{\sqrt{36+16}}{2}=\frac{\sqrt{52}}{2}=\frac{\sqrt{4\cdot 13}}{2}=\frac{2\sqrt{13}}{2}=\sqrt{13}$$

Bonus (10pts) Let $C=(x,y)$ be any point on the circle of radius r centered at the origin, and let $A=(-r,0)$ and $B=(r,0)$ be the endpoints of its diameter. Draw the picture and show that the triangle ABC is a right triangle. *(Hint: use the distance formula and the equation of the circle.)*



circle has equation

$x^2+y^2=r^2$

$$d(A,C)=\sqrt{(x-(-r))^2+(y-0)^2}=\sqrt{(x+r)^2+y^2}$$

$$d(B,C)=\sqrt{(x-r)^2+(y-0)^2}=\sqrt{(x-r)^2+y^2}$$

$$d(A,B)=2r$$

Check if $d(A,C)^2+d(B,C)^2=(2r)^2$

$$(x+r)^2+y^2+(x-r)^2+y^2=$$

$$=x^2+2xr+r^2+y^2+x^2-2xr+r^2+y^2=2x^2+2y^2+2r^2$$

$$=2\underbrace{(x^2+y^2)}_{=r^2 \text{ since } (x,y) \text{ is on circle.}}+2r^2=2r^2+2r^2=4r^2=(2r)^2$$