Topics in Topology — Handout
MAT 716, Spring 2014 — D. Ivanšić

Test Knowledge

Sections 6.1, 6.2, 4.3, 5.4, 8.2-8.4

Definitions

Cover, open cover of a set (6.1.1)
Compact space, subset (6.1.6)
Hausdorff space (6.1.14)
Bounded function (6.2.4)
Finite intersection property (6.2.14)
Product of sets (4.3.2)
Product topology (4.3.4)
Subbase of a topology (4.3.6)
Metric and metric topology (8.1.1, 8.1.8)
Uniformly continuous function (8.2.6)
Metrizability (8.2.11)
Sequence, convergent sequence in a topological space (8.3.1, 8.3.4)
Cauchy sequence (8.4.1)
Complete metric space (8.4.3)

Theorems

Theorem 6.1.12
Theorems 6.1.13, 6.1.19, 6.1.20, 6.1.21
Theorems 6.2.1, 6.2.2, 6.2.3
Theorems 6.2.5, 6.2.7
Theorems 6.2.11, 6.2.12, 6.2.13
Theorem 6.2.16
Theorems 4.3.11, 4.3.12, 4.3.14
Theorem 5.4.3
Theorem 8.2.1
Theorems 8.2.5, 8.2.7, 8.2.9
Theorems 8.2.13, 8.2.14, 8.2.15, 8.2.16
Theorems 8.3.9, 8.3.11, 8.3.12, 8.3.14
Lemma 8.4.6
Theorems 8.4.2, 8.4.4, 8.4.7

Proofs

Theorem 8.2.7
Theorem 8.3.14

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Test Knowledge

Sections Cantor Set, Quotient Topology, 0, 1.1

Definitions

Construction of the Cantor set
Quotient topology
Möbius strip, annulus
Pairings of a square that result in a torus and Klein bottle
Pairing of a 4g-gon that results in a genus-g surface
Sphere with crosscaps
Real projective space $\mathbf{R}\mathrm{P}^n$
Deformation retraction, deformation retract (p. 2))
Retraction, retract (p. 3)
Homotopy, homotopic maps, homotopy relative to a subspace (p. 3)
Homotopy equivalent spaces, homotopy type (p.3)
Contractible space (p.4)
Nullhomotopic map (p.4)
Path, path-connected space (p. 25)
Path homotopy (p. 25)
Fundamental group (p. 26)
Reparametrization of a path, inverse path (p. 27)
Induced homomorphism (p. 34)

Theorems

Theorem on bijection between Cantor set and certain sequences
Theorem on uncountability of Cantor set
Theorem on homeomorphism Cantor set $\leftrightarrow$ countable product of $\{0, 2\}$'s
Three equivalent ways of constructing a quotient topology
(map p, equivalence relation, partition)
Theorem on continuity of a map from a quotient space (theorem 1)
Theorem on construction of a map from a quotient space (theorem 2)
Lemma on identifications of a triangle and annulus that result in a Möbius strip
Klein bottle is homeomorphic to sphere with two crosscaps
Torus without disk with a crosscap is homeomorphic to disk with three crosscaps
$\mathbf{R}\mathrm{P}^2$ is homeomorphic to sphere with crosscap
Proposition 1.2
Proposition 1.3, 1.5
Theorem 1.7
Theorem 1.9
Proposition 1.12
Proposition 1.14, Corollay 1.16
Propositions 1.17, 1.18, Lemma 1.19

Proofs

Klein bottle is homeomorphic to sphere with two crosscaps
Torus without disk with a crosscap is homeomorphic to disk with three crosscaps
Proposition 1.14