College Algebra w.B.A. — Joysheet 9
MAT 120, Spring 2014 — D. Ivanšić

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Show all your work!

Solve the equations.

1. (8pts) $\log_2(x+1) = 3 + \log_2(x-4)$

$\log_2(x+1) - \log_2(x-4) = 3$

$\log_2 \frac{x+1}{x-4} = 3 \quad | \; 2^{\square}$

$\frac{x+1}{x-4} = 2^3 \quad | \cdot (x-4)$

$x+1 = 8(x-4)$

$x+1 = 8x - 32 \quad | -x+32$

$7x = 33$

$x = \frac{33}{7}$ (OK since it doesn't give 0 in denom)

2. (5pts) $4^{2-3x} = \left(\frac{1}{2}\right)^{3x+1}$

$(2^2)^{2-3x} = (2^{-1})^{3x+1}$

$2^{4-6x} = 2^{-3x-1}$

$4-6x = -3x-1 \quad | +6x+1$

$5 = 3x$

$x = \frac{5}{3}$

3. (7pts) $4^{3x-2} = 7^{2-x} \quad | \ln$

$\ln 4^{3x-2} = \ln 7^{2-x}$

$(3x-2)\ln 4 = (2-x)\ln 7$

$3x\ln 4 - 2\ln 4 = 2\ln 7 - x\ln 7$

$| + x\ln 7 + 2\ln 4$

$3x\ln 4 + x\ln 7 = 2\ln 7 + 2\ln 4$

$x(3\ln 4 + \ln 7) = 2(\ln 7 + \ln 4)$

$x = \frac{2(\ln 7 + \ln 4)}{3\ln 4 + \ln 7} = 1.0916668$

4. (14pts) The number of students enrolled at our fine school increased from 10,025 in 2008 to 10,832 in 2013. Assume the number of students follows the model $P(t) = y_0 b^t$.
a) Write the function describing the number $P(t)$ of students t years after 2008. What is the growth rate of MSU's student population?
b) Graph the function.
c) According to this model, when will Murray State have 14,000 students?

a) $y_0 = 10{,}025$

$P(t) = 10{,}025\, b^t$

$10{,}832 = P(5) = 10{,}025\, b^5$

$10{,}025\, b^5 = 10{,}832 \quad | \div 10{,}025$

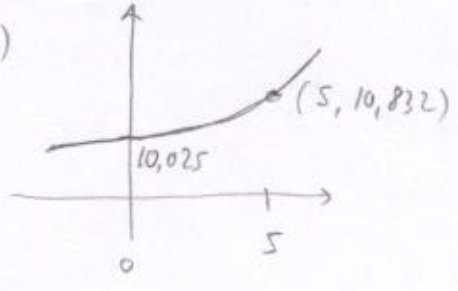
$b^5 = 1.0804\ldots$

$b = 1.0804^{\frac{1}{5}} = 1.015605$

$P(t) = 10{,}025 \cdot 1.015605^t$

Growth rate is $0.015605 = 1.5605\,\%$

b)



c) $P(t) = 14{,}000$

$10{,}025 \cdot 1.015^t = 14000 \quad | \div 10{,}025$

$1.015^t = 1.3965\ldots \quad | \ln$

$t \ln 1.015 = \ln 1.3965\ldots$

$t = \frac{\ln 1.3965\ldots}{\ln 1.015\ldots} = 21.568298$

Around 2029–2030

5. (8pts) Radiocarbon dating found that 80% of the original amount of carbon-14 is still present in a mummy sample. Assume half-life of carbon-14 is 5600 years. How old is the mummy?

$A(t) = y_0 \left(\frac{1}{2}\right)^{\frac{t}{5600}}$

$0.8\, y_0 = y_0 \left(\frac{1}{2}\right)^{\frac{t}{5600}} \mid \div y_0$

$0.8 = \left(\frac{1}{2}\right)^{\frac{t}{5600}} \mid \ln$

$\ln 0.8 = \ln\left(\frac{1}{2}\right)^{\frac{t}{5600}}$

$\frac{t}{5600} \ln\left(\frac{1}{2}\right) = \ln 0.8 \mid \cdot \frac{5600}{\ln 0.5}$

$t = \frac{\ln 0.8 \cdot 5600}{\ln 0.5} = 1802.797331$

Mummy is about 1803 years old.

6. (10pts) How much money should you deposit in a simple-interest account bearing 2.35% if you would like to have \$4000 in fifteen months? How much of the final \$4000 is from interest?

$A = P(1+rt)$

$4000 = P\left(1 + 0.0235 \cdot \frac{15}{12}\right)$

$4000 = P \cdot 1.029375 \mid \div 1.029375$

$P = \frac{4000}{1.029375} = 3885.85$

$I = A - P = 4000 - 3885.85$

$= 114.15$

7. (8pts) You can deposit \$1,000 into an account bearing 3.7% simple interest. How long will it take until you have \$1,500 in the account?

$A = P(1+rt)$

$1500 = 1000(1 + 0.037 \cdot t) \mid \div 1000$

$1.5 = 1 + 0.037t \mid -1$

$0.5 = 0.037t \mid \div 0.037$

$t = \frac{0.5}{0.037} = 13.513514$

About 13.5 years.