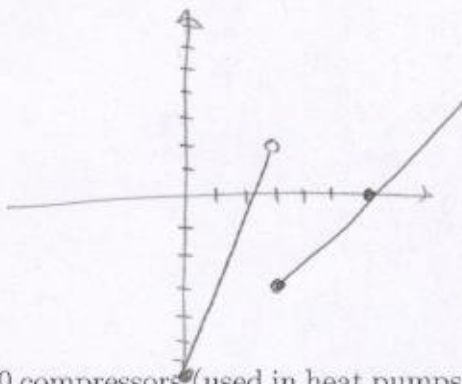**1.** (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 3x - 7, & \text{if } 0 \le x < 3 \\ x - 6, & \text{if } x \ge 3. \end{cases}$$

x	3x-7
0	-7
3	2

x	x-6
3	-3
6	0

2. (24pts) Suppose the cost to produce 1,000 compressors (used in heat pumps) is $1,242,000 and the cost to produce 5,000 compressors is $5,042,000. Assume the manufacturer can sell the compressors for $1,200.
a) Find the cost function, assuming it is linear.
b) What is the marginal cost of producing the 6001st compressor?
c) What is the average cost of producing 2,000 compressors? 4,000 compressors?
d) Write the revenue function for selling x compressors.
e) Write the profit function for selling x compressors.
f) What is the revenue of selling 3,000 compressors? The profit?
g) What is the break-even point in this example?

a) $(1,000, 1,242,000)$ are on the
$(5,000, 5,092,000)$ line

$m = \dfrac{5092000 - 1242000}{5000 - 1000} = \dfrac{3800,000}{4,000}$

$= \dfrac{3800}{4} = 950$

$y - 1242000 = 950(x - 1000)$

$y = 950x + 1,242,000 - 950,000$

$y = 950x + 292,000$

b) marginal cost = slope of line
$= 950$

c) $\dfrac{C(2000)}{2000} = \dfrac{950 \cdot 2000 + 292000}{2000} = 1096$

$\dfrac{C(4000)}{4000} = \dfrac{950 \cdot 4000 + 292000}{4000} = 1023$

d) $R(x) = 1200x$

e) $P(x) = R(x) - C(x) = 1200x - (950x + 292,000)$
$= 250x - 292,000$

f) $R(3000) = 1200 \cdot 3000 = 3,600,000$
$P(3000) = 250 \cdot 3000 - 292000 = 458,000$

g) $250x - 292000 = 0$

$x = \dfrac{292000}{250} = 1168$

3. (14pts) Suppose the supply and demand functions for a toaster oven are:
supply: $p = 1.4q$;   demand: $p = 80 - 2.3q$; p in dollars, q in some units.
a) Find the price if demand is 12 units.
b) Find the demand at price \$50. Find the supply at that price.
c) Graph the supply and demand curves on the same axes.
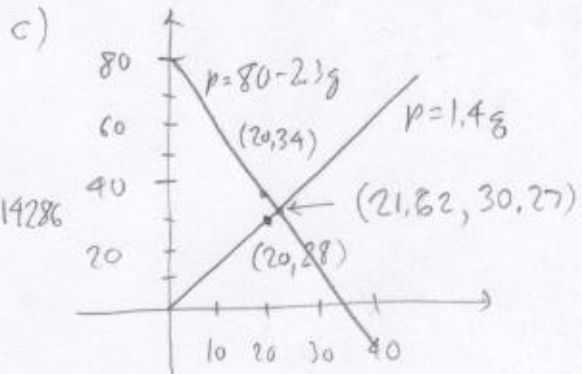d) Find the equilibrium price and equilibrium quantity for our example.

a) $p = 80 - 2.3 \cdot 12 = 52.4$

b) $80 - 2.3q = 50$

$30 = 2.3q$

$q = \frac{30}{2.3} = 13.043478$

Supply:
$50 = 1.4q$
$q = \frac{50}{1.4} = 35.714286$

c)

$p = 80 - 2.3q$
$(20, 34)$
$p = 1.4q$
$(21.62, 30.27)$
$(20, 28)$

d) $1.4q = 80 - 2.3q$ $|+2.3q$

$3.7q = 80$

$q = \frac{80}{3.7} = 21.621621$ equilibrium quantity

$p = 1.4 \cdot 21.621621 = 30.270270$ equilibrium price

4. (14pts) The quadratic function $f(x) = -x^2 - x + 20$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any. Find the y-intercept.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
d) Write the function in standard form.

a) y-int: $f(0) = 20$

x-int:
$-x^2 - x + 20 = 0$ $| \cdot (-1)$
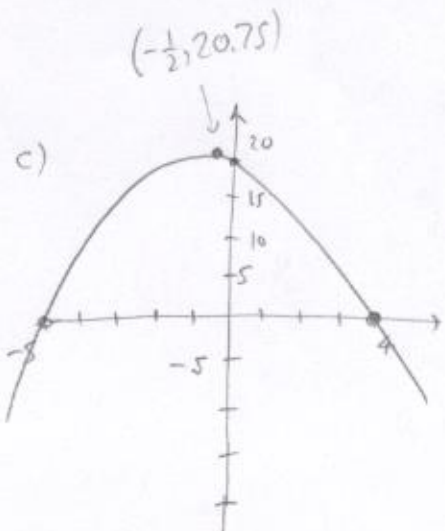$x^2 + x - 20 = 0$
$(x + 5)(x - 4) = 0$
$x = -5, 4$

b) $h = -\frac{-1}{2 \cdot (-1)} = -\frac{1}{2}$

$k = \left(-\frac{1}{2}\right)^2 - \left(-\frac{1}{2}\right) + 20$
$= \frac{1}{4} + \frac{1}{2} + 20$
$= 20.75$

c)

$\left(-\frac{1}{2}, 20.75\right)$

d) $f(x) = -\left(x - \left(-\frac{1}{2}\right)\right)^2 + 20.75$
$= -\left(x + \frac{1}{2}\right)^2 + 20.75$