College Algebra w.B.A.— Exam 4 MAT 120, Spring 2014 — D. Ivanšić

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_3 81 = 4$ $\qquad$ $\log_{25} \frac{1}{625} = -2$ $\qquad$ $\log_a \sqrt[7]{a^2} = \frac{2}{7}$ $\qquad$ $\log_{a^4} a^3 = \frac{3}{4}$ (think root)

$3^? = 81$ $\qquad$ $25^? = \frac{1}{625}$ $\quad 25^2 = 625$ $\qquad$ $a^? = \sqrt[7]{a^2} = a^{\frac{2}{7}}$ $\qquad$ $(a^4)^x = a^3 \quad 4x = 3, \ x = \frac{3}{4}$

2. (4pts) Use your calculator to find $\log_3 35$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_3 35 = \frac{\ln 35}{\ln 3} \approx 3.236217$

3. (8pts) If $\log_a 4 = 0.699795$ and $\log_a 5 = 0.812437$, find (show how you obtained your numbers):

$\log_a 20 = \log_a 4 + \log_a 5$
$= 0.699795 + 0.812437$
$= 1.512232$

$\log_a \frac{16}{25} = \log_a 16 - \log_a 25$
$= \log_a 4^2 - \log_a 5^2$
$= 2\log_a 4 - 2\log_a 5$
$= -0.225284$

$a = 7.25$
$a^{0.699795} = 4$
$a = 4^{\frac{1}{0.699795}}$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4(16x^4 \sqrt[3]{y^7}) = \log_4 16 + \log_4 x^4 + \log_4 y^{\frac{7}{3}}$

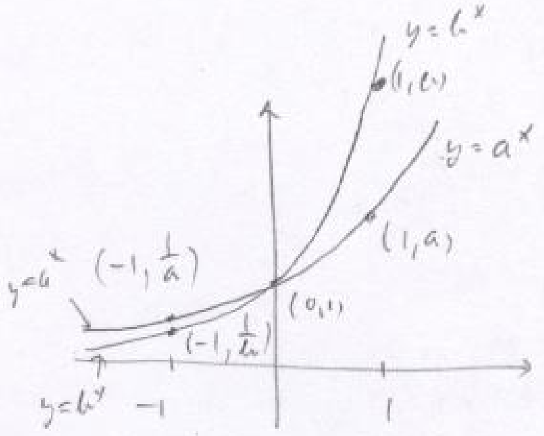
$= 2 + 4\log_4 x + \frac{7}{3}\log_4 y$

5. (6pts) Write as a single logarithm. Simplify if possible.

$2\ln(x^3y^5) - 3\ln(x^{-2}y^4) = \ln(x^3y^5)^2 - \ln(x^{-2}y^4)^3$

$= \ln \frac{(x^3y^5)^2}{(x^{-2}y^4)^3} = \ln \frac{x^6 y^{10}}{x^{-6} y^{12}} = \ln(x^{12}y^{-2}) = \ln \frac{x^{12}}{y^2}$

6. (8pts) Suppose $1 < a < b$. On the same set of axes, sketch $y = a^x$ and $y = b^x$. Indicate three points on each graph. Make sure the position of those two graphs with respect to each other is clear and accurate.



7. (10pts) How much money should you deposit in a simple-interest account bearing 4.5% if you would like to have \$5,000 in a year-and-a-half? How much of the final \$5000 is from interest?

$A = P(1+rt)$

$5000 = P(1+0.045 \cdot 1.5)$

$5000 = P \cdot 1.0675 \quad | \div 1.0675$

$P = 4683.84$

$I = A - P = 5000 - 4683.84$

$= 316.16$

8. (8pts) Paul borrowed \$2,000 and repaid it with \$2,250 after 10 months. What simple annual interest rate did this loan carry?

$A = P(1+rt)$

$2250 = 2000\left(1 + r \cdot \frac{10}{12}\right) \quad | \div 2000$

$1.125 = 1 + r \cdot \frac{5}{6} \quad | -1$

$0.125 = r \cdot \frac{5}{6} \quad | \cdot \frac{6}{5}$

$r = 0.125 \cdot \frac{6}{5} = 0.15$

It is 15% annually

Solve the equations.

9. (8pts) $27^{2x+1} = 9^{x-7}$

$(3^3)^{2x+1} = (3^2)^{x-7}$

$3^{6x+3} = 3^{2x-14}$

$6x+3 = 2x-14 \quad | -2x-3$

$4x = -17$

$x = -\frac{17}{4}$

10. (10pts) $\log_4(x+5) = 2 - \log_4(x-1)$

$\log_4(x+5) + \log_4(x-1) = 2$

$\log_4\left((x+5)(x-1)\right) = 2 \quad | 4^{-}$

$(x+5)(x-1) = 4^2$

$x^2+4x-5=16$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x=-7 \leftarrow$ gives negative in log, so not a solution

$\boxed{x=3}$ ok

11. (10pts) If an investment has a constant growth rate, its value after t years is described by the function $A(t) = y_0 b^t$. Suppose an investment has an 8% growth rate. How long until it doubles?

8% growth rate $\Rightarrow b = 1.08$

$A(t) = y_0 \cdot 1.08^t$

Need t when

$y_0 1.08^t = 2y_0 \quad | \div y_0$

$1.08^t = 2 \quad | \ln$

$\ln 1.08^t = \ln 2$

$t \ln 1.08 = \ln 2$

$t = \frac{\ln 2}{\ln 1.08} = 9.006468$

About 9 years

12. (14pts) The population of Growerton increased from 30,000 in 1998 to 45,000 in 2005. Assume the population follows the model $P(t) = y_0 b^t$.
a) Write the function describing the the population $P(t)$ of Growerton t years after 1998. What is the city's growth rate?
b) Graph the function.
c) According to this model, when will the city have 60,000 residents?

a) $y_0 = 30$

$P(t) = 30\, b^t$ (in thousands)

$45 = P(7) = 30\, b^7$

$30\, b^7 = 45 \quad | \div 30$

$b^7 = 1.5$

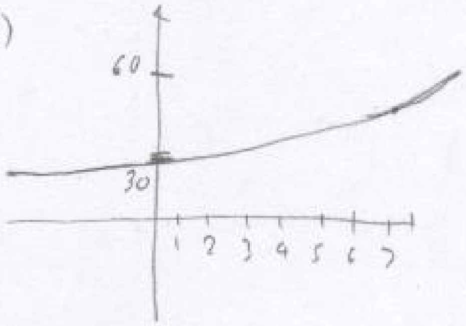
$b = 1.5^{\frac{1}{7}} = 1{,}059634$

Growth rate is $b - 1 = 0{,}059634$

i.e. $5{,}9634\%$

$P(t) = 30 \cdot 1{,}059634^t$

b)



exponential graph

c) $P(t) = 60$

$30 \cdot 1{,}059..^t = 60 \quad | \div 30$

$1.059..^t = 2 \quad | \ln$

$\ln 1{,}059..^t = \ln 2$

$t \ln 1{,}059.. = \ln 2 \qquad t = \frac{\ln 2}{\ln 1{,}059..} = 11.966579$

in about 12 years, so, in 2010.

Bonus (10pts) The life expectancy at birth of a person born in year x is approximately $f(x) = 17.6 + 12.8 \ln x$, where $x = 10$ corresponds to 1910.
a) What is the life expectancy at birth of a person born in 1960?
b) If life expectancy of a person at birth is 75, when were they born?

a) $1960 \Leftrightarrow x = 60$

$f(60) = 17.6 + 12.8 \ln 60 = 70.00761$

b) $f(x) = 75$

$17{,}6 + 12{,}8 \ln x = 75 \quad | -17{,}6$

$12.8 \ln x = 57.4 \quad | \div 12.8$

$\ln x = 4.484375 \quad | e^{-}$

$x = e^{4.484375} = 88.621545 \approx 89$

which corresponds to 1989.