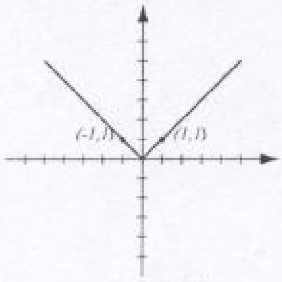
College Algebra w.B.A.— Exam 3
MAT 120, Spring 2014 — D. Ivanšić

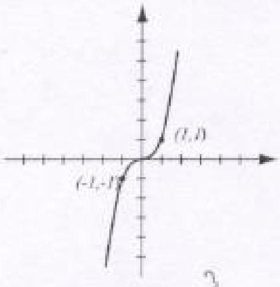
Name: Saul Ocean

Show all your work!

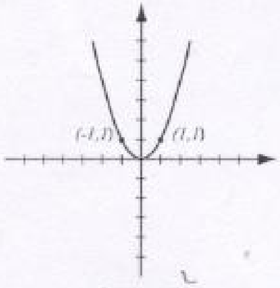
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



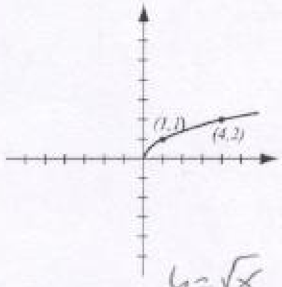
$y=|x|$



$y=x^3$



$y=x^2$



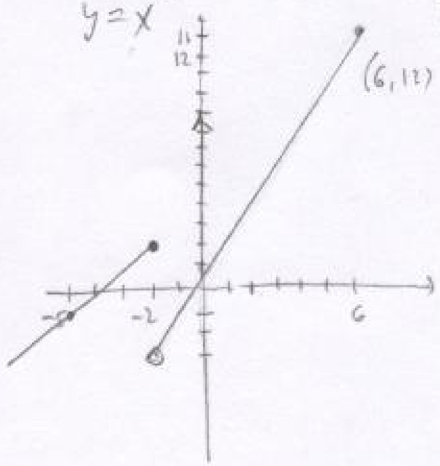
$y=\sqrt{x}$

2. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x)=\begin{cases} x+4, & \text{if } x\le -2 \\ 2x+1, & \text{if } -2<x\le 6. \end{cases}$$

x	$x+4$
-2	2
-5	-1

x	$2x+1$
-2	-3
6	13



3. (14pts) The quadratic function $f(x)=x^2-4x+7$ is given. Do the following without using the calculator.
a) Find the x-intercepts of its graph, if any. Find the y-intercept.
b) Find the vertex of the graph.
c) Sketch the graph of the function.
d) Write the function in standard form.

a) $x^2-4x+7=0$

$$x=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot1\cdot7}}{2\cdot1}$$

$$=\frac{4\pm\sqrt{-12}}{2}$$

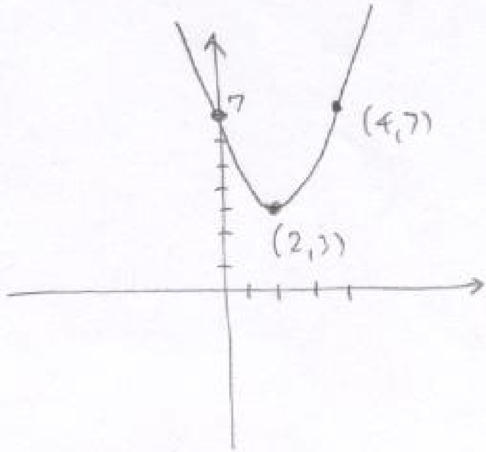
no real solutions, so no x-int.

$y=f(0)=7$

b) $h=-\frac{b}{2a}=-\frac{-4}{2\cdot1}=2$

$k=2^2-4\cdot2+7=3$

c)



d) $(x-2)^2+3$

4. (20pts) Suppose the cost to produce 3,000 mixers (used in the kitchen) is \$81,000 and the cost to produce 7,000 mixers is \$129,000. Tthe manufacturer can sell the mixers for \$17.
a) Find the cost function, assuming it is linear.
b) What is the average cost of producing 2,000 mixers? 5,000 mixers?
c) Write the revenue function for selling x mixers.
d) Write the profit function for selling x mixers.
e) What is the break-even point in this example?

a) (3000, 81,000) are on
(7,000, 129,000) line

$m = \frac{129000-81000}{7000-3000} = \frac{48000}{4000} = 12$

$y - 81,000 = 12(x-3000)$

$y = 12x - 36000 + 81000$
$= 12x + 45000$

b) $\frac{C(2000)}{2000} = \frac{12 \cdot 2000 + 45000}{2000} = 34.5$

$\frac{C(5000)}{5000} = \frac{12 \cdot 5000 + 45000}{5000} = 21$

c) $R(x) = 17x$

d) $P(x) = R(x) - C(x) = 17x - (12x + 45000)$
$= 5x - 45000$

e) $5x - 45000 = 0$

$x = \frac{45000}{5} = 9,000$.

5. (16pts) Suppose the supply and demand functions for some item are:
supply: $p = q^2 + q + 10$; demand: $p = 126 - 10q$; p in dollars, q in some units.
a) Find the price if demand is 3 units.
b) Find the demand at price \$30. Find the supply at price \$30.
c) Find the equilibrium price and equilibrium quantity for our example.

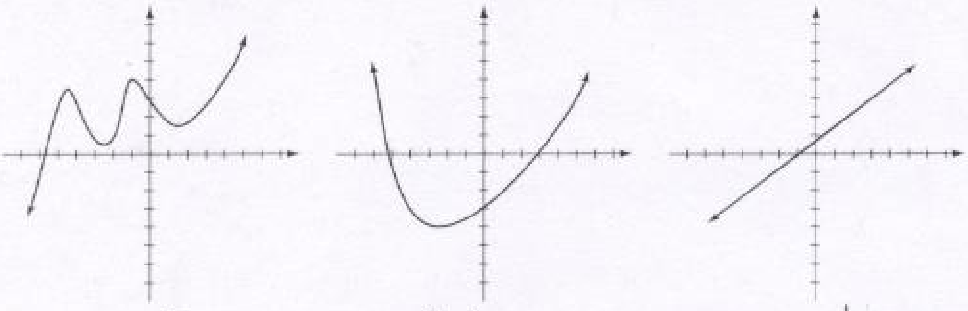
a) $q = 3$
$p = 126 - 10 \cdot 3 = 96$

b) $p = 30$
$30 = 126 - 10q$
$-96 = -10q$
$q = 9.6$

$p = 30$
$30 = q^2 + q + 10$
$q^2 + q - 20 = 0$
$(q-4)(q+5) = 0$
$q = 4, -5$ negative
$q = 4$.

c) supply = demand
$q^2 + q + 10 = 126 - 10q$
$q^2 + 11q - 116 = 0$
$q = \frac{-11 \pm \sqrt{11^2 - 4 \cdot (-1) 116}}{2 \cdot 1}$
$= \frac{-11 \pm \sqrt{121 + 464}}{2} = \frac{-11 \pm \sqrt{585}}{2}$
$=$ neg, 6.593387, $p = 126 - 10 \cdot 6.59...$
not a solution
$= 60.066134$
$= \$60.07$

6. (6pts) Under the graphs below, write 1, 2, 3, 4 or 5 if it could be the graph of a polynomial of degree 1, 2, 3, 4 or 5. More than one number per graph is possible.



5   2, 4   1

7. (14pts) Graph the polynomial $P(x) = (x+2)^2(x-2)^2(x+4)$ by following the guidelines.
a) Find the x-intercepts of the graph and the y-intercept.
b) What is the graph like for large $|x|$?
c) Sketch the graph of the polynomial on paper. Make sure scale is marked and all features you found in a), b) and d) are indicated.
d) Find the peaks and valleys of P.

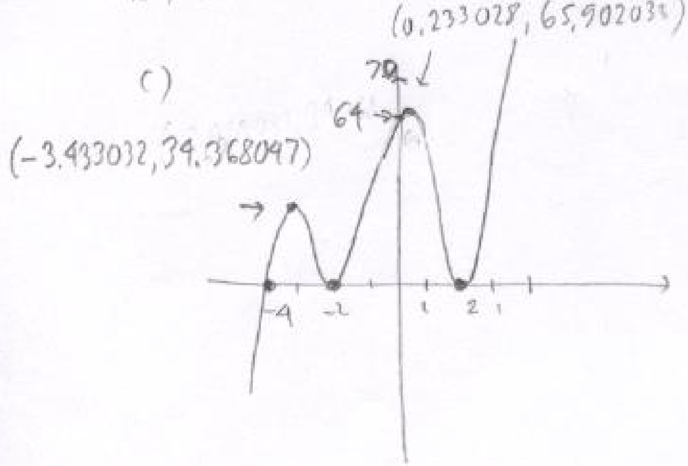
a) $(x+2)^2(x-2)^2(x+4)=0$

$x=-2, 2, -4$

$P(0)= 2^2(-2)^2\cdot 4 = 64$

b) Behaves like $x^2\cdot x^2\cdot x = x^5$

c)



d)
$(-2, 0)$ valleys
$(2, 0)$
$(0.233028, 65.902033)$
$(-3.433032, 34.368047)$

8. (14pts) Farmer Charles, who has 500 feet of fencing, wishes to enclose a rectangular field with the largest area next to a river. The side along the river does not need fencing.
a) Let x be the width of the enclosure. Find the length in terms of x.
b) Express the area of the enclosure as a function of x.
c) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the enclosure that has the greatest area and what is the greatest area possible?

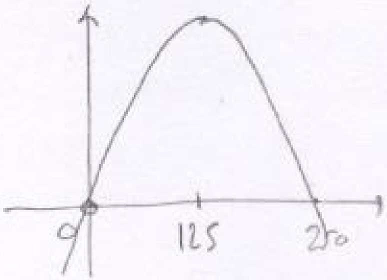
a) $2x+y=500$

$y=500-2x$

b) $A=x\cdot y=x(500-2x)=-2x^2+500x$

c) $x(500-2x)=0$

$x=0, 250$
(x-int of parabola)

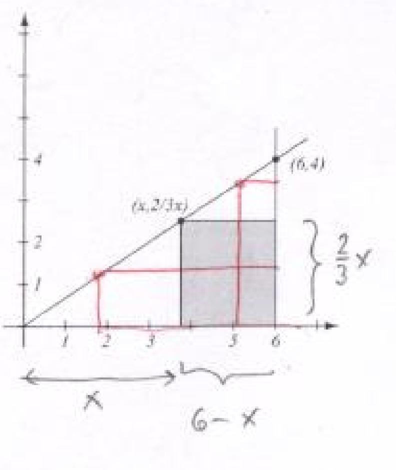


$h=\frac{0+250}{2}=125$

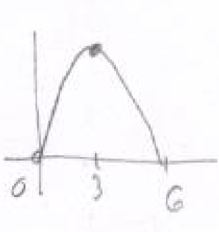
$k=125\cdot(500-2\cdot 125)$
$=125\cdot 250=31250$

dimensions: 125×250
max. area: $31,250\ ft^2$

Bonus. (10pts) Consider all rectangles whose two sides are on the lines $x=6$ and the x-axis, and one vertex is on the line $y=\frac{2}{3}x$, as in the picture.
a) Draw two more rectangles that fit this description.
b) Among all such rectangles, find the one with the greatest area. What are its dimensions?



b) The area of such a rectangle is
$A(x)=(6-x)\frac{2}{3}x=-\frac{2}{3}x^2+4x$



Max area occurs when $x=3$
Rectangle is 3×2, with area 6.

$h=\frac{0+6}{2}=3$

$k=(6-3)\cdot\frac{2}{3}\cdot 3=3\cdot 2=6$