

Factor the expressions:

1. (3pts)  $4x^2 - 25y^2 = (2x)^2 - (5y)^2 = (2x-5y)(2x+5y)$

2. (5pts)  $12x^2 + 4x - 5 = 12x^2 + 10x - 6x - 5 = 2x(6x+5) - (6x+5)$   
 $\text{prod} = -60 \quad 10, -6$   
 $\text{sum} = 4$   
 $= (2x-1)(6x+5)$

3. (5pts)  $9u^2 - 24uv + 16v^2 = (3u)^2 - 2 \cdot 3u \cdot 4v + (4v)^2 = (3u-4v)^2$

Simplify, showing intermediate steps.

4. (3pts)  $\sqrt{48} + \sqrt{12} = \sqrt{16 \cdot 3} + \sqrt{4 \cdot 3}$   
 $= 4\sqrt{3} + 2\sqrt{3} = 6\sqrt{3}$

5. (2pts)  $\sqrt[3]{72} = \sqrt[3]{8 \cdot 9} = 2\sqrt[3]{9}$

6. (8pts) Simplify.

$$\frac{2x+3}{x^2+x-6} - \frac{5}{x^2+6x+9} = \frac{(2x+3)(x+3) - 5(x-2)}{(x-2)(x+3)(x+3)} = \frac{2x^2+9x+9-5x+10}{(x-2)(x+3)(x+3)}$$

$$= \frac{2x^2+4x+19}{(x-2)(x+3)^2}$$

$\leftarrow$   $\begin{array}{l} \text{prod} = 38 \quad | \quad \begin{array}{l} \pm \\ 1, 38 \\ 2, 19 \end{array} \\ \text{sum} = 4 \quad | \quad \begin{array}{l} \text{no} \\ \text{no} \end{array} \end{array}$   
 doesn't factor

7. (8pts) Simplify. Express answers first in terms of positive exponents.

$$\frac{(125x^6y^{-\frac{1}{2}})^{\frac{1}{3}}}{(10x^3y^{\frac{3}{4}})^2} = \frac{125^{\frac{1}{3}} (x^6)^{\frac{1}{3}} (y^{-\frac{1}{2}})^{\frac{1}{3}}}{10^2 (x^3)^2 (y^{\frac{3}{4}})^2} = \frac{5x^2y^{-\frac{1}{6}}}{100x^6y^{\frac{3}{2}}} = \frac{x^{-4}y^{-\frac{5}{3}}}{20} = \frac{1}{20x^4y^{\frac{5}{3}}}$$

$-\frac{1}{6} - \frac{3}{2} = \frac{-1-9}{6} = -\frac{10}{6} = -\frac{5}{3}$

8. (5pts) Rationalize the denominator.

$$\frac{3\sqrt{5}-2}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{9\sqrt{5}-6-3\cdot 5+2\sqrt{5}}{3^2-\sqrt{5}^2} = \frac{11\sqrt{5}-21}{9-5} = \frac{11\sqrt{5}-21}{4}$$

Solve the equations.

9. (3pts)  $3(x-5)+2=5(x+4)$

$$3x-15+2=5x+20$$

$$3x-13=5x+20 \quad | -x-20$$

$$-33=2x$$

$$x = -\frac{33}{2}$$

10. (5pts)  $a+b+abc=ab+ac$  (solve for  $b$ )

$$b+abc-ab=ac-a \quad | -ab-a$$

$$(1+ac-a)b=ac-a$$

$$b = \frac{ac-a}{1+ac-a}$$

11. (5pts)  $|x-3|=|2x+1|$

$$x-3=2x+1 \quad \text{or} \quad x-3=-(2x+1)$$

$$-4=x \quad \text{or} \quad x-3=-2x-1$$

$$3x=2$$

$$x = \frac{2}{3}$$

$$x = -4 \quad \text{or} \quad x = \frac{2}{3}$$

12. (6pts)  $x^2+20x+7=5x-x^2 \quad | +x^2-5x$

$$2x^2+15x+7=0$$

$$x = \frac{-15 \pm \sqrt{15^2 - 4 \cdot 2 \cdot 7}}{2 \cdot 2}$$

$$= \frac{-15 \pm \sqrt{225-56}}{4}$$

$$= \frac{-15 \pm \sqrt{169}}{4} = \frac{-15 \pm 13}{4} = -7, -\frac{1}{2}$$

$$x = -7, -\frac{1}{2}$$

13. (14pts) A company makes large pots for plants. Its cost to make  $x$  thousand pots,  $0 \leq x \leq 30$ , is  $0.25x^2 + 11x + 34$  (also in thousands). Suppose the company can sell the pots for \$23 a piece.

a) Find the expressions for revenue and profit (in thousands) when selling  $x$  thousand axes.

b) Find the profit when 23,800 pots are produced.

c) How many pots are produced in order to make a profit of \$75,000?

a)  $R = 23x$

$$P = R - C = 23x - (0.25x^2 + 11x + 34)$$

$$P = -0.25x^2 + 12x - 34$$

b)  $x = 23.8$

$$P = -0.25 \cdot 23.8^2 + 12 \cdot 23.8 - 34$$

$$= 109.99$$

$$P = \$109,990$$

c)  $P = 75$

$$75 = -0.25x^2 + 12x - 34$$

$$-0.25x^2 + 12x - 109 = 0 \quad | \cdot (-1)$$

$$0.25x^2 - 12x + 109 = 0$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 0.25 \cdot 109}}{2 \cdot 0.25}$$

$$= \frac{12 \pm \sqrt{35}}{0.5} = 35.832160 \leftarrow \text{not in } (0, 30)$$

$$12.167840$$

12,168 pots produced for given profit

14. (14pts) You have 8% and 15% solutions of muriatic acid. How many liters of each solution is needed to get 10 liters of an 11% solution of muriatic acid? Write the meaning of your variable.

$$\begin{array}{c} x \\ \boxed{8\%} \end{array} + \begin{array}{c} 10-x \\ \boxed{15\%} \end{array} = \begin{array}{c} 10 \\ \boxed{11\%} \end{array}$$

$x = \text{liters of } 8\% \text{ sol.}$

$$0.08x + 0.15(10-x) = 0.11 \cdot 10$$

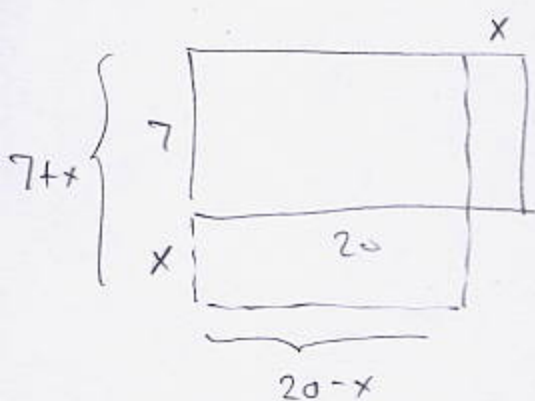
$$0.08x + 1.5 - 0.15x = 1.1 \quad | -1.5$$

$$-0.07x = -0.4$$

$$x = \frac{0.4}{0.07} = 5.714286 \text{ liters of } 8\%$$

$$10-x = 4.285714 \text{ liters of } 15\%$$

15. (14pts) A plot of land is 20 meters long and 7 meters wide. If we increase the width and decrease the length by the same amount we get a plot of area  $180\text{m}^2$ . Find this amount. Write the meaning of your variable.



$x =$  amount of increase of width

$$(7+x)(20-x) = 180$$

$$140 + 13x - x^2 = 180$$

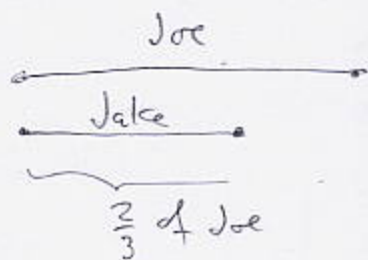
$$x^2 - 13x + 40 = 0$$

$$(x-8)(x-5) = 0$$

$$x = 5, 8$$

Increase width by either 5m (get  $12 \times 15$  rectangle)  
or 8m (get  $15 \times 12$  rectangle)

- Bonus** (10pts) Joe's tractor can travel 10mph faster than Jake's. Starting at the same time, Joe completed an entire journey when Jake completed  $\frac{2}{3}$  of this same journey. What are the speeds of Joe's and Jake's tractors? Write the meaning of your variable.



$r =$  Jake's speed

	dist.	rate	time
Jake	$\frac{2}{3}d$	$r$	$t$
	$d$	$r+10$	$t$

$$\frac{2}{3}d = rt$$

$$d = (r+10)t$$

$$\frac{2}{3}(r+10)t = rt \quad | \div t$$

$$\frac{2}{3}(r+10) = r$$

$$\frac{2}{3}r + \frac{20}{3} = r \quad | -\frac{2}{3}r$$

$$\frac{20}{3} = \frac{1}{3}r$$

$$r = 20 \text{ mph}$$

Jake's speed: 20 mph

Joe's speed: 30 mph