

1. (12pts) Simplify and write the answer so all exponents are positive:

a) $(2x^2y^{-4})^5(4x^{-3}y^{-7})^3 =$

b) $\frac{(6u^3v^{-5})^2}{(9u^2v^{-4})^3} =$

2. (4pts) Convert to scientific notation or a decimal number:

$3.32906 \times 10^5 =$

$0.00002387 =$

3. (8pts) Simplify and write in standard form:

a) $x^2(7x + 1) - (x - 4)(2x + 5) =$

b) $(3x + 5)(x^2 - 7x - 4) =$

4. (15pts) Use formulas to expand:

a) $(2x - 7)(2x + 7) =$

b) $(3x + 5y)^2 =$

c) $(5x - 2)^3 =$

5. (15pts) Factor the following. Use either a known formula or a factoring method.

a) $x^2 - 10x - 24 =$

b) $6x^2 - 29x - 5 =$

c) $64v^3 + 8 =$

6. (6pts) Write the following sets in interval notation. Then graph the interval.

$\{x \mid 4 \leq x < 13\}$

$\{x \mid x \geq -5\}$

$\{x \mid x < 3\}$

College Algebra — Joysheet 2
MAT 140, Fall 2014 — D. Ivanišić

Name: _____
Show all your work!

Solve the equations.

1. (3pts) $3(x - 4) + 2 = x + 2(3x - 1)$

2. (6pts) $x^2 - 16 = 5 - 4x$

3. (5pts) Solve the equation for b :

$$ac + bd = ad + bc$$

Simplify.

4. (8pts) $\frac{3x + 1}{2x^2 - 11x + 15} - \frac{x}{x^2 + x - 12} =$

5. (8pts) $\frac{\frac{a}{b} + \frac{b}{a}}{\frac{1}{b} - \frac{1}{a}} =$

Simplify, showing intermediate steps. Assume variables can be any real numbers.

6. (2pts) $\sqrt{75} =$

7. (4pts) $\sqrt{147x^7y^8} =$

8. (5pts) $\sqrt[3]{48x^4y^5}\sqrt[3]{20x^2y^3} =$

9. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to root notation.

$$\frac{\left(x^{-\frac{1}{2}}y^{\frac{3}{5}}\right)^{\frac{2}{3}}}{\left(x^{10}y^{\frac{1}{3}}\right)^{\frac{4}{5}}} =$$

10. (5pts) Simplify.

$$(\sqrt{2} + 4)(3\sqrt{8} - 5) =$$

11. (6pts) Rationalize the denominator.

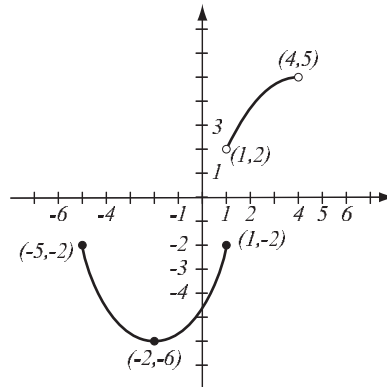
$$\frac{\sqrt{3} - 2\sqrt{5}}{\sqrt{5} - \sqrt{3}}$$

1. (8pts) Use the distance formula to find out whether the triangle with vertices $A = (-1, -3)$, $B = (3, 0)$ and $C = (0, 2)$ is an isosceles triangle.

2. (8pts) Write the equation of the circle whose diameter has endpoints $(4, -3)$ and $(-2, -1)$. Sketch the circle.

3. (8pts) Use the graph of the function f at right to answer the following questions.

- a) Find $f(-2)$ and $f(1)$.
- b) What is the domain of f ?
- c) What is the range of f ?
- d) What are the solutions of the equation $f(x) = -2$?



4. (12pts) The function $f(x) = (x - 9)\sqrt{x + 4} + 7$ is given.

a) Use your calculator to accurately its graph. Draw the graph here, and indicate units on the axes.

b) Find all the x - and y -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (12pts) Find the domain of each function and write it using interval notation.

$$f(x) = \frac{\sqrt[5]{3x + 24}}{2x - 5}$$

$$g(x) = \frac{\sqrt{x}}{x^2 + 5x - 24}$$

6. (12pts) Let $g(x) = \frac{2x - 7}{x^2 - 3x - 4}$. Find the following (simplify where appropriate).

$$g(-3)$$

$$g(4)$$

$$g(2u)$$

$$g(x - 3)$$

1. (6pts) Find the equation of the line (in form $y = mx + b$) that passes through points $(-2, -3)$ and $(-2, 5)$.

2. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through point $(-3, -1)$ and is perpendicular to the line that has x -intercept 4 and y -intercept 3. Draw both lines.

3. (8pts) Draw the quadrangle with vertices $A = (-4, -1)$, $B = (0, -3)$, $C = (2, 1)$, $D = (-2, 3)$. Then use slopes to show that this quadrangle is a rectangle. Do not use the Pythagorean theorem.

4. (4pts) The number of highway fatalities in the US has decreased from 43,510 in 2005 to 32,784 in 2010. Find the average rate of change in the number of highway fatalities from 2005 to 2010. What are the units for the average rate?

5. (12pts) Ann is the customer of an innovative cell-phone company that charges on actual monthly phone usage. In one month, when she used 122 minutes, she paid \$15.66. In another month, when she used 214 minutes, she paid \$20.72.

a) Assuming that monthly cost $C(x)$ is a linear function of minutes used x , write a formula for $C(x)$.

b) How much does Ann pay if she uses 0 minutes? What is the meaning of this number?

c) What is the meaning of the slope in this example?

6. (20pts) A researcher is studying the number of intersections with traffic lights and the number of traffic accidents in small towns in order to model the relationship between lights and accidents. The table shows the data, where I is the number of intersections, and A is the number of accidents in a year. Solve the problems below with accuracy 6 decimal points.

a) Draw the scatterplot of the data. Does the relationship look linear?

b) Use two points in the scatterplot to get an equation of a line that models the relationship between I and A . Draw the line on the graph.

c) Use your calculator to find the “line of best fit” for the data. Draw the line on the graph.

d) Find the coefficient of correlation r . How strong is the linear relationship between I and A ?

e) What annual number of accidents does the researcher expect in a small town that has 30 intersections with traffic lights?

I	A
11	383
14	370
21	302
25	231
27	223
35	150

Solve the inequalities. Write your solution in interval notation.

1. (5pts) $-3 \leq 2x - 3 < 15$

2. (7pts) $3x - 7 \leq 3$ or $2x + 5 > 16$

3. (6pts) Find the domain of the function $f(x) = \frac{\sqrt{9 - 4x}}{3x + 2}$ (in interval notation).

4. (14pts) Elena, an apparel store employee, can be paid on one of two plans:

A) Salary of \$540 per month, plus a commission of 12% of sales, or

B) Salary of \$820 per month, plus a commission of 7% of sales over \$1,500.

Assuming Elena can always sell more than \$1,500, for what level of sales is plan A better?

5. (14pts) On a drive to Hopkinsville you take the scenic route and drive at average speed 50 mph. On your return, you take a road that is 10 miles longer, but since you can drive at average speed 60 mph, it takes you 2 minutes shorter than the trip to Hopkinsville. How far is Hopkinsville from your starting point?

6. (14pts) How many liters of a 4% solution of sulphuric acid must be mixed with how many liters of a 10% solution of sulphuric acid in order to get 5 liters of an 8% solution of sulphuric acid?

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = 2x^3 + 4x^2 - 9x - 3$. Draw the graph here, and indicate units on the axes.
- a) Find the local maxima and minima for this function.
- b) State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let $f(x) = \frac{1}{3x+5}$, $g(x) = \frac{x}{2x-1}$. Find the following (simplify where possible):

$$(f - g)(4) =$$

$$(fg)(7) =$$

$$\frac{f}{g}(x) =$$

$$(f \circ g)(2) =$$

$$(g \circ f)(x) =$$

The domain of $(f + g)(x)$ in interval notation

3. (8pts) Consider the function $h(x) = \frac{4}{3x+1}$. Find functions f and g so that $h(x) = f(g(x))$. Find two different solutions to this problem, neither of which is the “stupid” one.

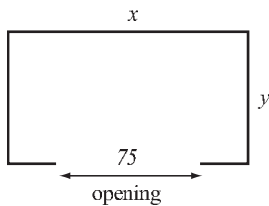
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -2x + 4, & \text{if } x < 3 \\ 5 - x, & \text{if } 3 \leq x < 5. \end{cases}$$

5. (14pts) An airport wishes to build a hangar for planes that is to have area 10,000 square feet, and has to have a 75-foot wide opening for a door on one side (see picture). To minimize cost, the total length of walls has to be as small as possible.

a) Express the total length of walls of the hangar as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the hangar that has the smallest total wall length?



1. (21pts) For the following functions, determine algebraically whether they odd, even, or neither. Then use the calculator to draw their graphs on paper and verify your conclusions.

$$f(x) = x^3 - x + 1$$

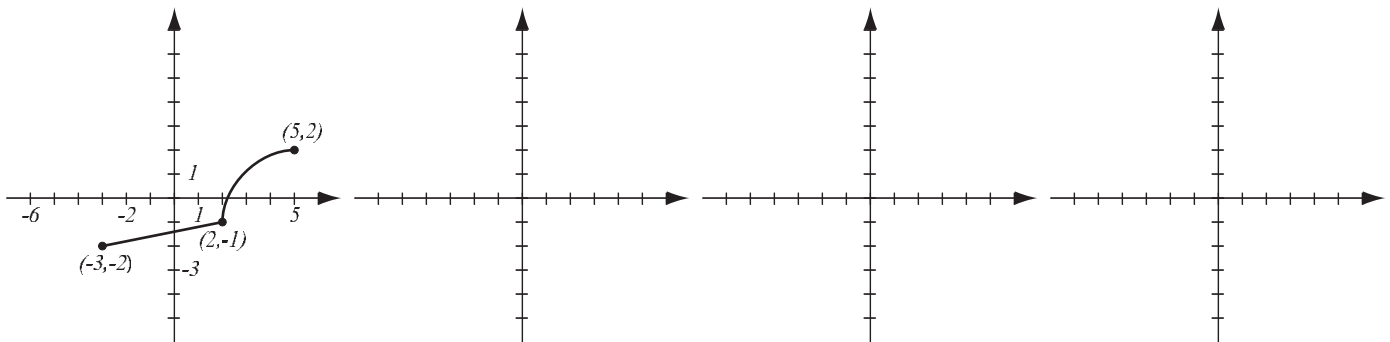
$$g(x) = x^4 - 3x^2 + 4$$

$$h(x) = x^5 - 7x^3 + 10x$$

2. (16pts) Using transformations, draw the graphs of $f(x) = 3 + \sqrt[3]{2x}$ and $g(x) = -\frac{1}{2}(x+4)^2$. Explain how you transform graphs of basic functions in order to get the graphs of f and g .

3. (9pts) Write the equation for the function whose graph has the following characteristics:
- shape of $y = x^3$, shifted down 4 units
 - shape of $y = \frac{1}{x}$, stretched vertically by factor 2, then shifted left 3 units
 - shape of $y = \sqrt{x}$, reflected about the x -axis, then stretched horizontally by factor 3, then shifted up 5 units.

4. (14pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x + 3)$, $2f(x)$ and $-f(2x)$ and label all the relevant points.



Simplify, so that the answer is in form $a + bi$.

1. (4pts) $(3 - 2i)(2 + 3i) =$

2. (6pts) $\frac{4 - 7i}{5 + 4i} =$

3. (4pts) Simplify and justify your answer.

$i^{114} =$

4. (6pts) Use the discriminant to determine how many x -intercepts (no need to find them) the graphs of the following quadratic functions have.

$f(x) = 3x^2 - 5x + 4$

$f(x) = -x^2 - 6x - 9$

$f(x) = \sqrt{2}x^2 + \sqrt{3}x - \sqrt{8}$

5. (8pts) Solve the equation: $4x^4 + 4x^2 - 35 = 0$

6. (6pts) Solve by completing the square.

$x^2 - 14x + 30 = 0$

7. (12pts) The quadratic function $f(x) = -x^2 + 3x + 9$ is given. Do the following without using the calculator.

- a) Find the x -intercepts of its graph, if any. Find the y -intercept.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

8. (14pts) On Juanita's farm, children playing in a rectangular enclosure 110 by 80 meters cannot, due to range limitations, reach each other with toy walkie-talkies from any two places in the play area. Therefore, Juanita will make it smaller by reducing the 110-meter side by twice the amount that the 80-meter side is reduced. If the range of the walkie-talkies is 100 meters, by how much will Juanita reduce the 110- and 80-meter sides? (Note that this is equivalent to having the diagonal of the new enclosure be 100 meters, since the diagonal represents the greatest distance between any two points in a rectangle.)

1. (4pts) Solve the equation.

$$|3x - 7| = 11$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x - 4| \geq 5$$

$$|4x + 12| < 9$$

Solve the equations:

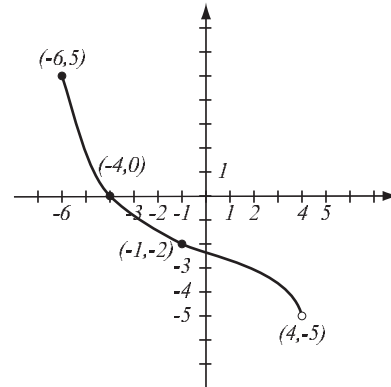
3. (8pts) $\frac{2x}{x-5} + \frac{7}{x-1} = \frac{x^2 + 3x}{x^2 - 6x + 5}$

4. (8pts) $\sqrt{33-x} - \sqrt{40-3x} - 1 = 0$

5. (14pts) A ball is thrown upwards from the ground with initial velocity 45 ft/sec.
- Write the function that describes the height of the ball in feet t seconds after release.
 - When does the ball reach its greatest height, and what is that height?
 - When is the ball at height 25 feet?

6. (14pts) A small orchard has ripening pears. At the start of the selling season, 180 pounds of pears were picked and in storage. Every day after that, 30 pounds of pears are picked and added to storage. Suppose the value of a pound of pears is \$4 at the start and decreases 10 cents per day after that.
- Express the value of all the pears in storage as a function of the number of days x since the start of the selling season. What is the domain of this function?
 - Sketch the graph of the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). How many days after the start of the selling season is the value of the pears in storage maximal and what is this value?

1. (6pts) The graph of a function f is given.
- Is this function one-to-one? Justify.
 - If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points.



2. (12pts) Let $f(x) = \frac{2x - 1}{3x + 7}$. Find the formula for f^{-1} . Find the domain and range of f .

3. (8pts) Evaluate without using the calculator:

$$\log_2 128 = \quad \log_4 \frac{1}{64} = \quad \log_{25} 125 = \quad \log_b \sqrt[7]{b^3} =$$

4. (4pts) Use your calculator to find $\log_{12} 0.5$ with accuracy 6 decimal places. Show how you obtained your number.

5. (6pts) If you invest \$3,000 in an account bearing 3.33%, compounded monthly, how much is in the account in 42 months?

6. (3pts) Find the domain of $f(x) = \log_9(3x - 1)$.

7. (7pts) The cost per household for taking the US census is modeled by the function $C(t) = 15.5202(1.0508)^t$, where t is the number of years since 1970.

a) Find the per-household census cost in 1990. According to the model, what will be the per-household cost in 2020?

b) Use the intersect feature on the calculator to estimate in what year the per-household cost will go past \$200.

8. (14pts) Using transformations, draw the graphs of $f(x) = 3 - 2^{-x}$ and $g(x) = \frac{1}{2} \log(x - 5)$. Explain how you transform graphs of basic functions in order to get the graphs of f and g .

1. (5pts) If $\log_a 7 = 0.94148$ and $\log_a 9 = 1.063072$, find (show how you obtained your numbers):

$$\log_a 63 =$$

$$\log_a \frac{7}{81} =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_6 (216x^3y^8) =$$

$$\log_4 \sqrt[8]{\frac{64x^7y^{-5}}{x^2y^3}} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$\frac{1}{4} \log(16x^3) - 2 \log(7y^{\frac{5}{4}}) - \log(x^{\frac{7}{4}}) =$$

$$2 \log_a(x+2) + 3 \log_a(x-4) - 3 \log_a(x^2 - 2x - 8) =$$

Solve the equations.

4. (5pts) $27^{2-5x} = 3^{2x-7}$

5. (7pts) $2^{4x+3} = 3^{5x-4}$

6. (8pts) $e^{2x} - 14 = 3e^x + 14$

7. (12pts) The 2000 and 2010 censuses recorded Nashville, TN as having approximately 569,000 and 627,000 people, respectively. Assume Nashville's population grows exponentially.

a) Write the function describing the number $P(t)$ of people t years after 2000. Then find the exponential growth rate of Nashville's population.

b) Graph the function.

c) According to this model, when will the population reach 800,000?