College Algebra — Joysheet 11
MAT 140, Fall 2014 — D. Ivanšić

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Show all your work!

1. (5pts) If $\log_a 7 = 0.94148$ and $\log_a 9 = 1.063072$, find (show how you obtained your numbers):

$\log_a 63 = \log_a(7\cdot 9)$
$= \log_a 7 + \log_a 9$
$= 2.004552$

$\log_a \frac{7}{81} = \log_a 7 - \log_a 9^2$
$= \log_a 7 - 2\log_a 9$
$= -1.184664$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_6(216x^3y^8) = \log_6 216 + \log_6 x^3 + \log_6 y^8$
$= 3 + 3\log_6 x + 8\log_6 y$

$\log_4 \sqrt[8]{\frac{64x^7y^{-5}}{x^2y^3}} = \log_4\left(64x^5y^{-8}\right)^{\frac{1}{8}} = \frac{1}{8}\log_4(64x^5y^{-8})$
$= \frac{1}{8}\left(\log_4 64 + \log_4 x^5 + \log_4 y^{-8}\right)$
$= \frac{1}{8}(3 + 5\log_4 x - 8\log_4 y) = \frac{3}{8} + \frac{5}{8}\log_4 x - \log_4 y$

3. (12pts) Write as a single logarithm. Simplify if possible.

$\frac{1}{4}\log(16x^3) - 2\log(7y^{\frac{5}{4}}) - \log(x^{\frac{7}{4}}) = \log(16x^3)^{\frac{1}{4}} - \log(7y^{\frac{5}{4}})^2 - \log x^{\frac{7}{4}}$
$= \log(2x^{\frac{3}{4}}) - \log 49y^{\frac{5}{2}} - \log x^{\frac{7}{4}} = \log\frac{2x^{\cancel{\frac{3}{4}}}}{49y^{\frac{5}{2}}x^{\cancel{\frac{7}{4}}\,1}} = \log\frac{2}{49xy^{5/2}}$

$2\log_a(x+2) + 3\log_a(x-4) - 3\log_a(x^2-2x-8) =$

$= \log_a(x+2)^2 + \log_a(x-4)^3 - \log_a(\underbrace{x^2-2x-8}_{(x+2)(x-4)})^3$

$= \log\frac{(x+2)^2(x-4)^3}{((x+2)(x-4))^3} = \log\frac{(x+2)^2(x-4)^3}{(x+2)^3(x-4)^3} = \log\frac{1}{x+2}$

Solve the equations.

4. (5pts) $27^{2-5x} = 3^{2x-7}$

$$(3^3)^{2-5x} = 3^{2x-7}$$
$$3^{6-15x} = 3^{2x-7}$$
$$6-15x = 2x-7$$
$$13 = 17x$$
$$x = \frac{13}{17}$$

5. (7pts) $2^{4x+3} = 3^{5x-4}$ $\quad | \ln$

$$\ln 2^{4x+3} = \ln 3^{5x-4}$$
$$(4x+3)\ln 2 = (5x-4)\ln 3$$
$$4x\ln 2 + 3\ln 2 = 5x\ln 3 - 4\ln 3$$
$$4x\ln 2 - 5x\ln 3 = -3\ln 2 - 4\ln 3$$
$$x(4\ln 2 - 5\ln 3) = -3\ln 2 - 4\ln 3$$
$$x = \frac{-3\ln 2 - 4\ln 3}{4\ln 2 - 5\ln 3} = \frac{3\ln 2 + 4\ln 3}{5\ln 3 - 4\ln 2}$$
$$x = 2.379693$$

6. (8pts) $e^{2x} - 14 = 3e^x + 14$

let $u = e^x$, $e^{2x} = u^2$

$$u^2 - 14 = 3u + 14$$
$$u^2 - 3u - 28 = 0$$
$$(u-7)(u+4) = 0$$
$$u = 7, -4$$

$e^x = 7$ $\quad$ $e^x = -4$

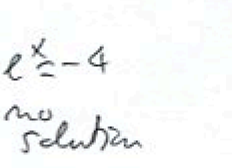
$x = \ln 7$ $\quad$ no solution

7. (12pts) The 2000 and 2010 censuses recorded Nashville, TN as having approximately 569,000 and 627,000 people, respectively. Assume Nashville's population grows exponentially.
a) Write the function describing the number $P(t)$ of people t years after 2000. Then find the exponential growth rate of Nashville's population.
b) Graph the function.
c) According to this model, when will the population reach 800,000?

a) $P(t) = 569e^{kt}$ (in thousands) $\quad P(t) = 569e^{0.00970661t}$

$$627 = 569e^{k\cdot 10}$$
$$\frac{627}{569} = e^{k\cdot 10} \quad | \ln$$
$$\ln\frac{627}{569} = k\cdot 10$$
$$k = \frac{\ln\frac{627}{569}}{10} = 0.00970661$$

b)



c) $569e^{0.009\_t} = 800$

$$e^{0.009\_t} = \frac{800}{569}$$
$$0.009\_t = \ln\frac{800}{569}$$
$$t = \frac{\ln\frac{800}{569}}{0.00970661} = 35.103014$$

Approximately in year 2035