| College Algebra — Joysheet 8 |
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| MAT 140, Fall 2014 — D. Ivanšić |

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*Show all your work!*

Simplify, so that the answer is in form $a + bi$.

**1.** (4pts) $(3-2i)(2+3i) = 6+9i-4i-6\underbrace{i^2}_{=-1} = 12+5i$

**2.** (6pts) $\dfrac{4-7i}{5+4i} = \dfrac{4-7i}{5+4i}\cdot\dfrac{5-4i}{5-4i} = \dfrac{20-35i-16i+28i^2}{5^2-(4i)^2} = \dfrac{-8-51i}{25+16}$

$= -\dfrac{8}{41} - \dfrac{51}{41}i$

**3.** (4pts) Simplify and justify your answer.

$i^{114} = \underset{\uparrow}{i^{112}}\, i^2 = 1\cdot i^2 = -1$

divisible by 4

**4.** (6pts) Use the discriminant to determine how many $x$-intercepts (no need to find them) the graphs of the following quadratic functions have.

$f(x) = 3x^2-5x+4$

$(-5)^2 - 4\cdot 3\cdot 4$

$= 25-48 = -23 < 0$

no x-int

Use $b^2-4ac$

$f(x) = -x^2-6x-9$

$(-6)^2 - 4(-1)(-9)$

$36-36 = 0$

one x-int

$f(x) = \sqrt{2}x^2+\sqrt{3}x-\sqrt{8}$

$\sqrt{3}^2 - 4\cdot\sqrt{2}(-\sqrt{8})$

$= 3 + 4\sqrt{16} = 19 > 0$

two x-int

**5.** (8pts) Solve the equation: $4x^4+4x^2-35=0$

Let $u = x^2$

$4u^2+4u-35=0$

$u = \dfrac{-4 \pm \sqrt{4^2-4\cdot 4\cdot(-35)}}{2\cdot 4}$

$u = \dfrac{-4\pm\sqrt{16+560}}{8} = \dfrac{-4\pm\sqrt{576}}{8} = \dfrac{-4\pm 24}{8}$

$= -\dfrac{28}{8}, \dfrac{20}{8} = -\dfrac{7}{2}, \dfrac{5}{2}$

$x^2 = -\dfrac{7}{2}$ $\quad x = \pm\sqrt{\dfrac{7}{2}}\,i$

$x^2 = \dfrac{5}{2}$ $\quad x = \pm\sqrt{\dfrac{5}{2}}$

**6.** (6pts) Solve by completing the square.

$x^2-14x+30=0 \quad | +7^2$

$x^2 - 2\cdot x\cdot 7 + 7^2 + 30 = 7^2$

$(x-7)^2 + 30 = 49 \quad | -30$

$(x-7)^2 = 19$

$x-7 = \pm\sqrt{19}$

$x = 7\pm\sqrt{19}$

**7.** (12pts) The quadratic function $f(x) = -x^2 + 3x + 9$ is given. Do the following without using the calculator.

a) Find the $x$-intercepts of its graph, if any. Find the $y$-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) x-int

$$-x^2+3x+9=0 \quad \cdot(-1)$$

$$x^2-3x-9=0$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4\cdot1\cdot(-9)}}{2\cdot1}$$

$$=\frac{3\pm\sqrt{9+36}}{2}=\frac{3\pm\sqrt{45}}{2}$$

$$=\frac{3\pm3\sqrt5}{2}=\begin{matrix}4.854102\\-1.854102\end{matrix}$$

y-int:

$$f(0)=9$$

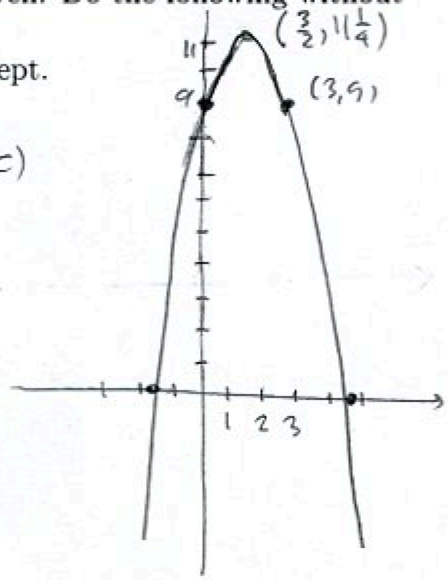
b) vertex

$$h=-\frac{3}{2\cdot(-1)}=\frac{3}{2}$$

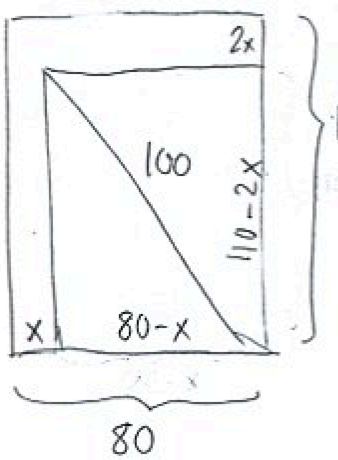
$$k=f\left(\frac{3}{2}\right)=-\frac{9}{4}+\frac{9}{2}+9$$

$$=\frac{9}{4}+9=\frac{45}{4}=11\frac{1}{4}$$

c)




**8.** (14pts) On Juanita's farm, children playing in a rectangular enclosure 110 by 80 meters cannot, due to range limitations, reach each other with toy walkie-talkies from any two places in the play area. Therefore, Juanita will make it smaller by reducing the 110-meter side by twice the amount that the 80-meter side is reduced. If the range of the walkie-talkies is 100 meters, by how much will Juanita reduce the 110- and 80-meter sides? (Note that this is equivalent to having the diagonal of the new enclosure be 100 meters, since the diagonal represents the greatest distance between any two points in a rectangle.)




$$(80-x)^2+(110-2x)^2=100^2$$

$$6400-160x+x^2+12100-440x+4x^2=10000$$

$$5x^2-600x+8500=0 \quad |\div 5$$

$$x^2-120x+1700=0$$

$$x=\frac{-(-120)\pm\sqrt{(-120)^2-4\cdot1\cdot1700}}{2}=\frac{120\pm\sqrt{14400-6800}}{2}$$

$$=\frac{120\pm\sqrt{7600}}{2}=\frac{120\pm20\sqrt{19}}{2}=60\pm10\sqrt{19}$$

$4\cdot19\cdot100$

$$=\begin{matrix}103.588989\\16.411011\end{matrix}$$

$103.5.. > 80$, so cannot be a solution.

Reduce

80 m side by 16.411011

110 m side by 32.822021