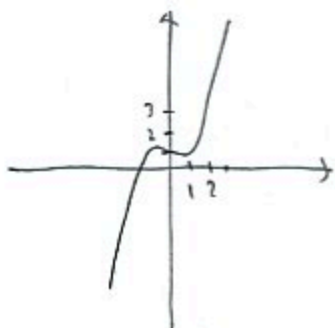


1. (21pts) For the following functions, determine algebraically whether they odd, even, or neither. Then use the calculator to draw their graphs on paper and verify your conclusions.

$$f(x) = x^3 - x + 1$$

$$\begin{aligned} f(-x) &= (-x)^3 - (-x) + 1 \\ &= -x^3 + x + 1 \neq f(x) \\ &\neq -f(x) \end{aligned}$$

neither

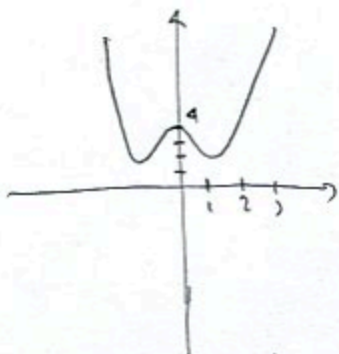


not symmetric

$$g(x) = x^4 - 3x^2 + 4$$

$$\begin{aligned} g(-x) &= (-x)^4 - 3(-x)^2 + 4 \\ &= x^4 - 3x^2 + 4 = g(x) \end{aligned}$$

even

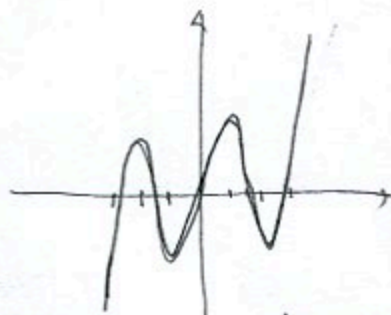


symmetric wrt.
y-axis

$$h(x) = x^5 - 7x^3 + 10x$$

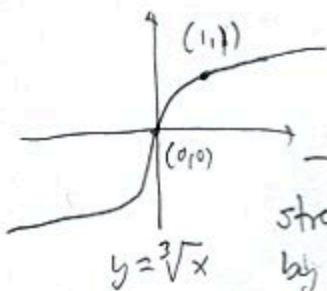
$$\begin{aligned} h(-x) &= (-x)^5 - 7(-x)^3 + 10(-x) \\ &= -x^5 - 7(-x^3) - 10x \\ &= -x^5 + 7x^3 - 10x = -h(x) \end{aligned}$$

odd

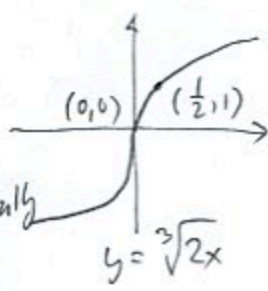


symmetric wrt
origin

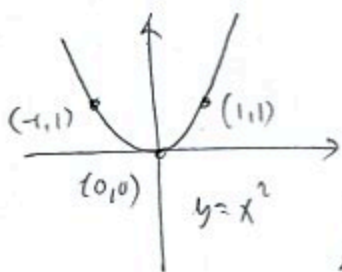
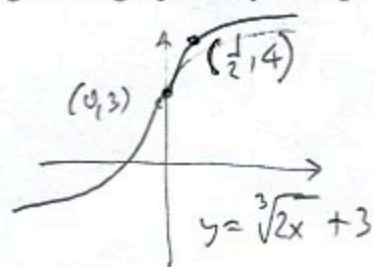
2. (16pts) Using transformations, draw the graphs of $f(x) = 3 + \sqrt[3]{2x}$ and $g(x) = -\frac{1}{2}(x+4)^2$. Explain how you transform graphs of basic functions in order to get the graphs of f and g .



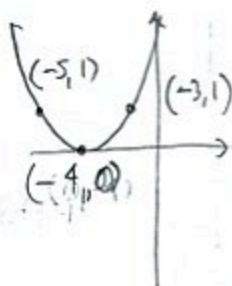
stretch horizontally
by factor $\frac{1}{2}$



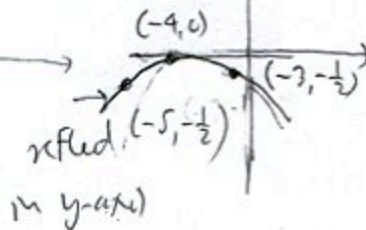
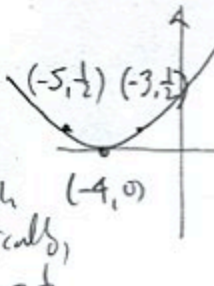
shift
up 3



shift
left 4



stretch
vertically,
Factor = $\frac{1}{2}$



3. (9pts) Write the equation for the function whose graph has the following characteristics:

a) shape of $y = x^3$, shifted down 4 units

b) shape of $y = \frac{1}{x}$, stretched vertically by factor 2, then shifted left 3 units

c) shape of $y = \sqrt{x}$, reflected about the x -axis, then stretched horizontally by factor 3, then shifted up 5 units.

$$a) f(x) = x^3 - 4$$

$$b) f(x) = \frac{2}{x+3}$$

$$x \mapsto 2 \cdot \frac{1}{x} \mapsto 2 \cdot \frac{1}{x+3}$$

$$c) f(x) = -\sqrt{\frac{1}{3}x} + 5$$

$$\sqrt{x} \rightarrow -\sqrt{x} \rightarrow -\sqrt{\frac{1}{3}x} \rightarrow -\sqrt{\frac{1}{3}x} + 5$$

4. (14pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(x+3)$, $2f(x)$ and $-f(2x)$ and label all the relevant points.

