College Algebra — Joysheet 6
MAT 140, Fall 2014 — D. Ivanšić

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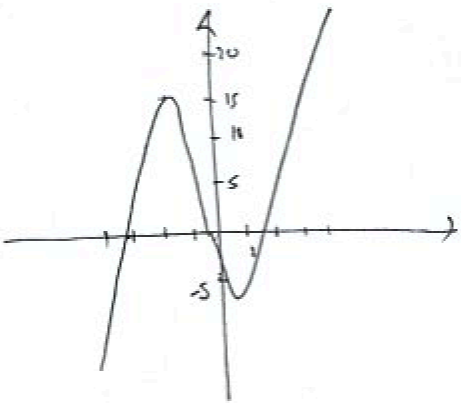
Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = 2x^3 + 4x^2 - 9x - 3$. Draw the graph here, and indicate units on the axes.
a) Find the local maxima and minima for this function.
b) State the intervals where the function is increasing and where it is decreasing.



a) $f(-2.061102) = -15.030778$ is a local max
$f(0.727765) = -6.660408$

b) Increasing on
$(-\infty, -2.061102)$ and $(0.727765, \infty)$
Decreasing on $(-2.061102, 0.727765)$

2. (20pts) Let $f(x) = \dfrac{1}{3x+5}$, $g(x) = \dfrac{x}{2x-1}$. Find the following (simplify where possible):

$(f-g)(4) = f(4) - g(4) = \dfrac{1}{17} - \dfrac{4}{7} = \dfrac{7-68}{119} = -\dfrac{61}{119}$

$(fg)(7) = f(7)\cdot g(7) = \dfrac{1}{26}\cdot\dfrac{7}{13} = \dfrac{7}{338}$

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{1}{3x+5}}{\frac{x}{2x-1}} = \dfrac{2x-1}{x(3x+5)}$

$(f\circ g)(2) = f(g(2)) = f\left(\dfrac{2}{3}\right) = \dfrac{1}{7}$

$(g\circ f)(x) = g(f(x)) = g\left(\dfrac{1}{3x+5}\right) = \dfrac{\frac{1}{3x+5}}{2\cdot\frac{1}{3x+5} - 1}\cdot\dfrac{3x+5}{3x+5} = \dfrac{1}{2-(3x+5)} = \dfrac{1}{-3x-3} = -\dfrac{1}{3x+3}$

The domain of $(f+g)(x)$ in interval notation

Domain of f:
can't have $3x+5=0$
$3x = -5$
$x = -\frac{5}{3}$

Domain of g:
can't have $2x-1=0$
$2x = 1$
$x = \frac{1}{2}$

$-\frac{5}{3}$ domain of f
$\frac{1}{2}$ domain of g
$-\frac{5}{3}$ $\frac{1}{2}$ domain of $f+g$ (intersection)

Domain of $f+g$: $(-\infty, -\frac{5}{3}) \cup (-\frac{5}{3}, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

3. (8pts) Consider the function $h(x) = \frac{4}{3x+1}$. Find functions f and g so that $h(x) = f(g(x))$. Find two different solutions to this problem, neither of which is the "stupid" one.

$g(x) = 3x+1$ $\qquad$ $g(x) = 3x$

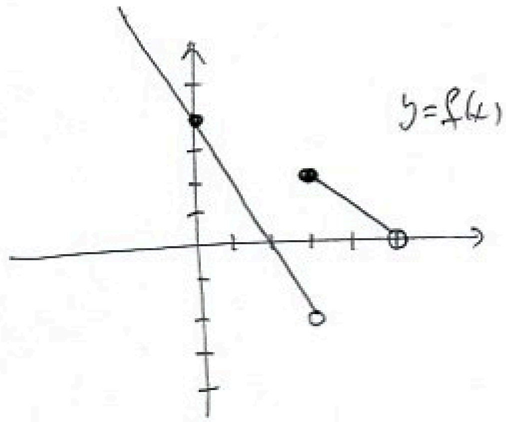
$f(x) = \frac{4}{x}$ $\qquad$ $f(x) = \frac{4}{x+1}$

4. (8pts) Sketch the graph of the piecewise-defined function:

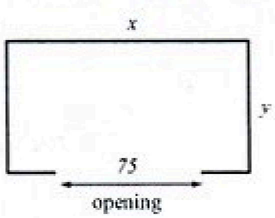
$$f(x) = \begin{cases} -2x+4, & \text{if } x < 3 \\ 5-x, & \text{if } 3 \le x < 5. \end{cases}$$

x	$-2x+4$
3	-2
0	4

x	$5-x$
3	2
5	0



5. (14pts) An airport wishes to build a hangar for planes that is to have area 10,000 square feet, and has to have a 75-foot wide opening for a door on one side (see picture). To minimize cost, the total length of walls has to be as small as possible.
a) Express the total length of walls of the hangar as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the hangar that has the smallest total wall length?



a) $\ell(x) = 2x + \frac{20000}{x} - 75$

x has to be bigger than opening

$x \geq 75$ domain

(no upper bound on x)

$\ell = x + 2y + x - 75$

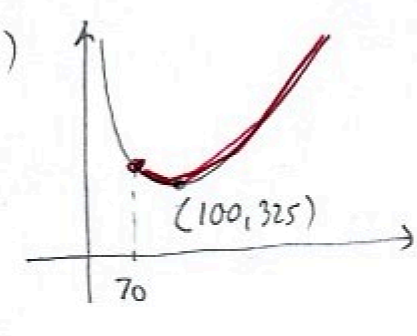
$= 2x + 2y - 75$

$xy = 10000$

$y = \frac{10000}{x}$

$\ell(x) = 2x + 2 \cdot \frac{10000}{x} - 75$

b)



Minimal wall length is $\ell(100) = 325$, occurs for hangar with dimensions

100×100

x $\qquad$ $y = \frac{10000}{x}$