

Solve the equations.

1. (3pts) $3(x-4) + 2 = x + 2(3x-1)$

$$\begin{aligned} 3x - 12 + 2 &= x + 6x - 2 \\ 3x - 10 &= 7x - 2 \quad | -3x + 2 \\ -8 &= 4x \quad | \div 4 \\ x &= -2 \end{aligned}$$

2. (6pts) $x^2 - 16 = 5 - 4x \quad | +4x - 5$

$$\begin{aligned} x^2 + 4x - 21 &= 0 \\ (x+7)(x-3) &= 0 \\ x &= -7, 3 \end{aligned}$$

3. (5pts) Solve the equation for b :

$$\begin{aligned} ac + bd &= ad + bc \quad | -bc - ac \\ bd - bc &= ad - ac \\ b(d-c) &= ad - ac \leftarrow a(d-c) \\ b &= \frac{a(d-c)}{d-c} = a \end{aligned}$$

Simplify.

4. (8pts) $\frac{3x+1}{2x^2-11x+15} - \frac{x}{x^2+x-12} = \frac{3x+1}{(2x-5)(x-3)} - \frac{x}{(x+4)(x-3)}$

prod = 30
sum = -11

$$\begin{aligned} \frac{3x+1}{(2x-5)(x-3)} - \frac{x}{(x+4)(x-3)} &= \frac{(3x+1)(x+4) - x(2x-5)}{(2x-5)(x-3)(x+4)} \\ &= \frac{3x^2 + 13x + 4 - (2x^2 - 5x)}{(2x-5)(x-3)(x+4)} \\ &= \frac{x^2 + 18x + 4}{(2x-5)(x-3)(x+4)} \end{aligned}$$

prod = 4
sum = 18
doesn't factor
no such integers

5. (8pts) $\frac{\frac{a}{b} + \frac{b}{a}}{\frac{1}{b} - \frac{1}{a}} = \frac{\frac{a \cdot a + b \cdot b}{ab}}{\frac{a-b}{ab}} = \frac{a^2 + b^2}{ab} \cdot \frac{ab}{a-b} = \frac{a^2 + b^2}{a-b}$ ← can't factor

Simplify, showing intermediate steps. Assume variables can be any real numbers.

6. (2pts) $\sqrt{75} = \sqrt{3 \cdot 25} = \boxed{5\sqrt{3}}$

7. (4pts) $\sqrt{147x^7y^8} = \sqrt{3 \cdot 49 \cdot x^6 \cdot x \cdot y^8} = \sqrt{3 \cdot 7^2 \cdot (x^3)^2 (y^4)^2 x} = \boxed{7|x^3|y^4\sqrt{3x}}$

8. (5pts) $\sqrt[3]{48x^4y^5} \sqrt[3]{20x^2y^3} = \sqrt[3]{\underset{3 \cdot 2}{6 \cdot 8} \cdot 4 \cdot 5 x^6 y^8} = \sqrt[3]{8 \cdot 8 \cdot 3 \cdot 5 (x^2)^3 (y^2)^3 y^2} = \boxed{4x^2y^2\sqrt[3]{15y^2}}$

9. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to root notation.

$$\frac{(x^{-\frac{1}{2}}y^{\frac{3}{5}})^{\frac{2}{3}}}{(x^{10}y^{\frac{1}{3}})^{\frac{4}{5}}} = \frac{x^{-\frac{1}{2} \cdot \frac{2}{3}} y^{\frac{3}{5} \cdot \frac{2}{3}}}{x^{10 \cdot \frac{4}{5}} y^{\frac{1}{3} \cdot \frac{4}{5}}} = \frac{x^{-\frac{1}{3}} y^{\frac{2}{5}}}{x^8 y^{\frac{4}{15}}} = x^{-\frac{1}{3}-8} y^{\frac{2}{5}-\frac{4}{15}} = x^{-\frac{25}{3}} y^{\frac{2}{15}}$$

$$= \frac{y^{\frac{2}{15}}}{x^{\frac{25}{3}}} = \frac{\sqrt[15]{y^2}}{\sqrt[3]{x^{25}}}$$

10. (5pts) Simplify.

$$(\sqrt{2} + 4)(3\sqrt{8} - 5) = 3\sqrt{16} - 5\sqrt{2} + 12\overset{4 \cdot 2}{\sqrt{8}} - 20$$

$$= 12 - 5\sqrt{2} + 24\sqrt{2} - 20$$

$$= -8 + 19\sqrt{2}$$

11. (6pts) Rationalize the denominator.

$$\frac{\sqrt{3} - 2\sqrt{5}}{\sqrt{5} - \sqrt{3}} \cdot \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{\sqrt{15} + \sqrt{3}^2 - 2\sqrt{5}^2 - 2\sqrt{15}}{\sqrt{5}^2 - \sqrt{3}^2} = \frac{-\sqrt{15} + 3 - 10}{5 - 3} = \frac{-7 - \sqrt{15}}{2}$$