College Algebra — Exam 1
MAT 140, Fall 2014 — D. Ivanšić

Name: _____

Show all your work!

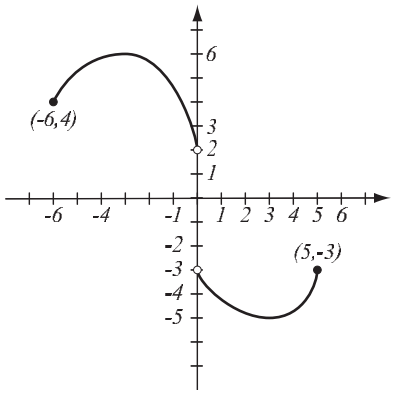
1. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find $f(3)$ and $f(0)$.

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = 4$?



2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3 - 10x - 17$. Draw the graph here, and indicate units on the axes. Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

3. (4pts) Convert to scientific notation or a decimal number:

$0.0000347 =$ $1.593 \times 10^4 =$

Use formulas to expand:

4. (4pts) $(4x + 5)^2 =$

5. (4pts) $(2x - u^2)(2x + u^2) =$

6. (6pts) Factor: $8x^3 - 125 =$

Simplify, showing intermediate steps. Assume variables can be any real numbers.

7. (2pts) $\sqrt[3]{108} =$

8. (5pts) $\sqrt{125x^7y^4} =$

9. (8pts) Simplify.

$$\frac{x-5}{3x^2-x-10} - \frac{2x}{x^2+3x-10} =$$

10. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to radical notation.

$$\frac{\left(x^3y^{-\frac{1}{2}}\right)^{\frac{3}{4}}}{\left(x^{\frac{2}{3}}y^4\right)^{\frac{1}{4}}} =$$

11. (6pts) Rationalize the denominator.

$$\frac{4-5\sqrt{3}}{\sqrt{3}+2}$$

12. (5pts) Solve the equation for t.

$c(a + bt) = d$

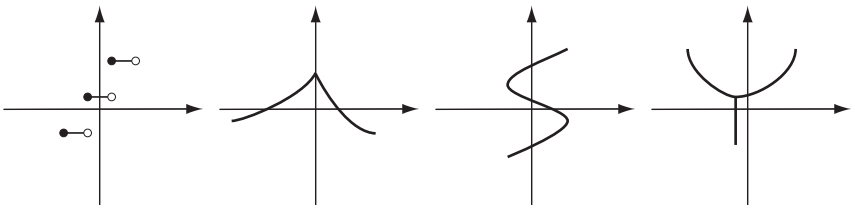
13. (8pts) Find the domain of the function $f(x) = \dfrac{1 + \sqrt{x}}{x^2 + 2x - 8}$ and write it using interval notation.

14. (10pts) Let $g(x) = (x^2 + 2)\sqrt{3 - x}$. Find the following (simplify where appropriate).

$g(-1)$ $g(8)$

$g(\sqrt{a})$ $g(x - 1)$

15. (4pts) Which of the following graphs are graphs of functions (yes/no)?



16. (8pts) A circle is centered at $(-3, 4)$ and passes through the origin.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

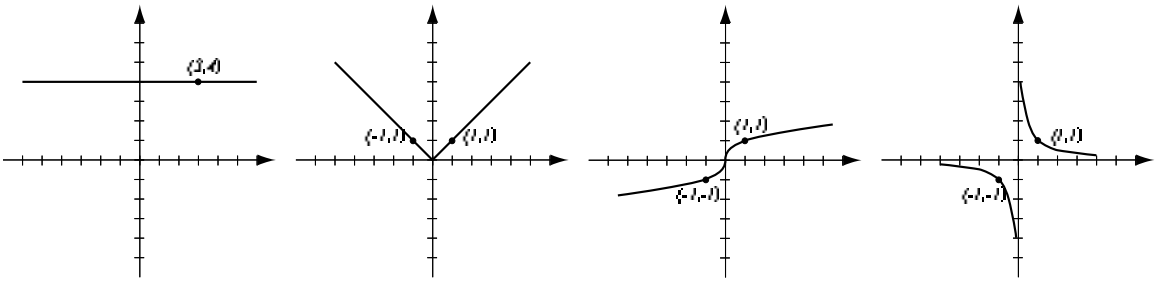
Bonus (10pts) Find points on the x-axis whose distance to point $(3, 2)$ is $\sqrt{29}$. *Hint: what form do coordinates of points on the x-axis have?*

College Algebra — Exam 2
MAT 140, Fall 2014 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through $(1, 2)$ and is perpendicular to the line $2x + 5y = 3$. Draw both lines.

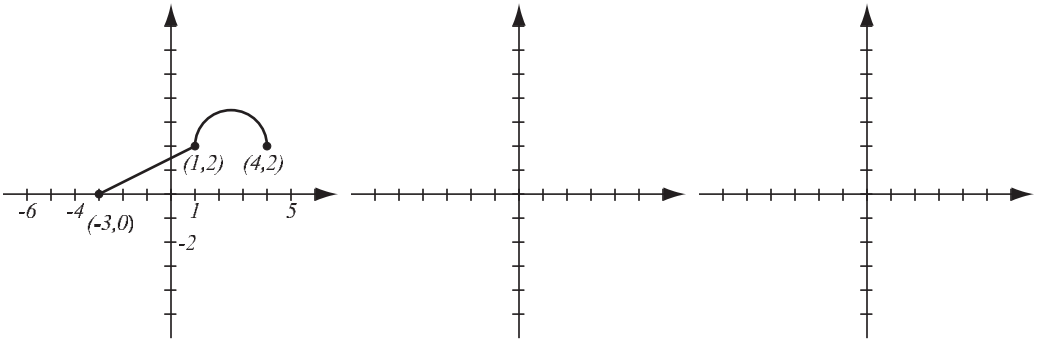
3. (5pts) Solve the inequality and write your solution in interval notation.

$3 \leq 3x - 1 < 7$

4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x - 3, & \text{if } -2 < x < 3 \\ 4 - x, & \text{if } x \geq 3 \end{cases}$$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x-3)$ and $f(-x)+1$ and label all the relevant points.



6. (14pts) Let $f(x) = \sqrt{2x+1}$, $g(x) = x^2 + 3$.

Find the following (simplify where possible):

$(f+g)(0) =$ $(fg)(x) =$

$(f \circ g)(-1) =$ $(g \circ f)(x) =$

The domain of f in interval notation

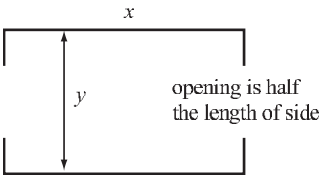
7. (4pts) Consider the function $h(x) = \sqrt[3]{x^2 - 2x + 4}$. Find functions f and g, neither of which is the "stupid" one, so that $h(x) = f(g(x))$.

8. (17pts) Let $f(x) = x^5 - 6x^3$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Then state how the graph supports your conclusion.
c) Find the local maxima and minima for this function.
d) State the intervals where the function is increasing and where it is decreasing.

9. (10pts) Prices for apples at two orchards are: Fufu Farms charges \$25 packing and 67 cents per pound, while Old McDonald's charges \$10 packing and 73 cents per pound. For which quantities of apples is Fufu Farms the better deal?

10. (14pts) Alison and Mitch bicycle along the same road. It takes Mitch 30 minutes to travel the road. Alison leaves 6 minutes after Mitch, but gets to the end of the road at the same time as Mitch because she travels 1.5 mph faster than Mitch.
a) What are the speeds of the cyclists?
b) How long is the road?

Bonus. (14pts) A trucking company wishes to build a service garage for trucks that is to have area 6400 square feet, and has openings on two sides that are half the length of the sides (see picture). To minimize cost, the total length of walls has to be as small as possible.
a) Express the total length of walls of the garage as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the garage that has the smallest total wall length?



College Algebra — Exam 3
MAT 140, Fall 2014 — D. Ivanšić

Name: _____

Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $3i(5 - i) + 3 - 2i =$

2. (6pts) $\dfrac{3 + 4i}{4 - 3i} =$

3. (4pts) Simplify and justify your answer.

$i^{155} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 12x + 41 = 0$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x + 3| \geq 7$

6. (6pts) Let $f(x)$ be some polynomial of degree 4.
a) State the maximum number of x-intercepts of the graph of f.
b) State the maximum number of turning points on the graph of f.
c) Can the range of f be $(-\infty, \infty)$? Why or why not?

7. (12pts) The quadratic function $f(x) = -x^2 + 4x + 21$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

Solve the equations:

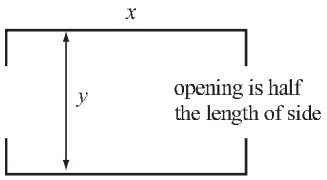
8. (8pts) $\dfrac{x+5}{x+3} + \dfrac{8}{x^2+2x-3} = \dfrac{4}{x-1}$

9. (8pts) $\sqrt{4t+5} + \sqrt{t+5} = 3$

10. (14pts) The polynomial $f(x) = (x+5)(x-2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) One side of a rectangle is 4 meters longer than the other. If the shorter side is tripled and the longer side extended by 1 meter, the resulting rectangle has area 63 m^2 greater than the original one. What are the dimensions of the original rectangle?

12. (14pts) A trucking company wishes to build a service garage for trucks that has openings on two sides that are half the length of the sides (see picture). They have enough money for 1200 feet of walls and wish to maximize the area of the service garage.
a) Express the area of the garage as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the garage that has the biggest possible area?



Bonus. (10pts) Follow the steps below to find the point on the line $y = 2x + 1$ that is closest to the point $(3, 4)$. (Sketch the line and the point.)
a) Write the function that represents the square of the distance (avoids a square root) from a point (x, y) on the line to the point $(3, 4)$ and express it in terms of x.
b) Graph the function in order to find the minimum and state the point on the line that is closest to $(3, 4)$.

College Algebra — Exam 4
MAT 140, Fall 2014 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_3 81 =$ $\log_5 \frac{1}{125} =$ $\log_a \sqrt[7]{a^3} =$ $\log_{\sqrt{b}} b^3 =$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_7 17$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 12 = c$ and $\log_a 5 = d$, express in terms of c and d:

$\log_a 60 =$ $\log_a \frac{144}{125} =$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

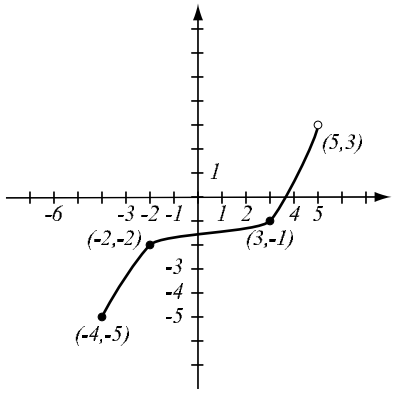
$\log_7 \frac{49x^3}{\sqrt[3]{y^8}} =$

5. (12pts) Write as a single logarithm. Simplify if possible.

$2\log(x^3y^4) - 5\log(x^2y^3) =$

$3\ln(x^2+7x-18) - 2\ln(x+9) - 4\ln(x-2) =$

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



7. (9pts) Let $f(x) = \dfrac{3x+1}{4x-1}$, $x \geq 0$.
a) Find the formula for f^{-1}.
b) Find the range of f.

8. (6pts) Using transformations, draw the graph of $f(x) = e^{-x} + 3$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

9. (6pts) Find the domain of the function $f(x) = \log_2(4x + 5) + \log_3(2 - 7x)$ and write it in interval notation.

10. (8pts) How much should you invest in an account bearing 4.02%, compounded quarterly, if you wish to have \$10,000 in five years?

Solve the equations.

11. (8pts) $2^{x+1} = 3^{1-x}$

12. (10pts) $\log_3(x - 2) + \log_3(x + 6) = 2$

13. (12pts) The population of Orlando, FL was 128,000 in 1980 and 238,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 1980. Graph it on paper.
b) Find the predicted population in the year 2015.

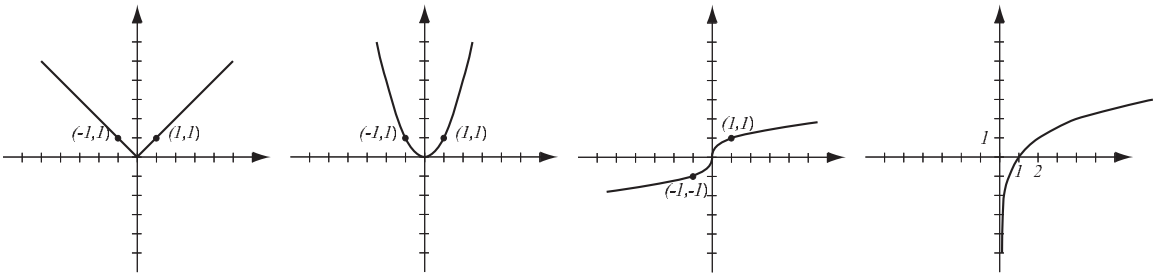
Bonus (10pts) Let $f(x) = x^2 - 6x$, considered for $x \leq 3$.
a) Sketch the graph of f and verify that the function is one-to-one.
b) Find the formula for the inverse of this function.

College Algebra — Final Exam
MAT 140, Fall 2014 — D. Ivanšić

Name: _____

Show all your work!

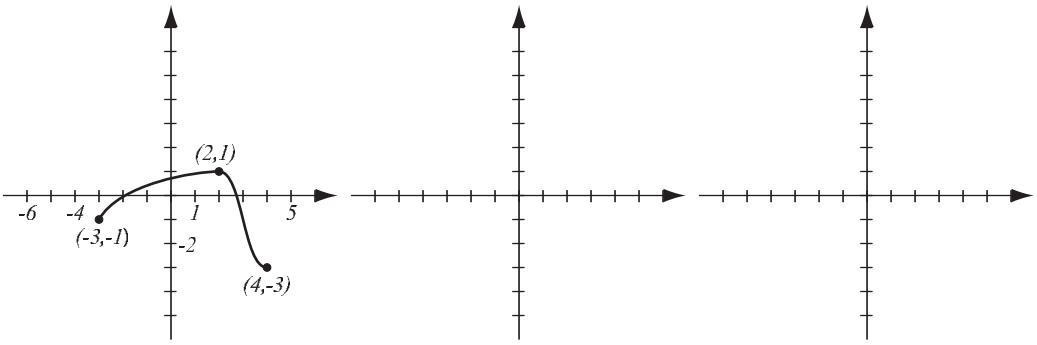
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (5pts) Find the equation of the line (in form $y = mx + b$) that passes through $(3, 1)$ and $(-2, 5)$.

3. (6pts) Is the triangle with vertices $A = (-3, 1)$, $B = (7, 0)$ and $C = (0, 5)$ a right triangle? Use either the distance formula or slopes of perpendicular lines to find out.

4. (8pts) The graph of the function f is given below. On separate graphs, sketch the graphs of the functions $f(x + 3)$ and $-2f(x)$. Label all the relevant points.



5. (10pts) Use the graph of the function f at right to answer the following questions.

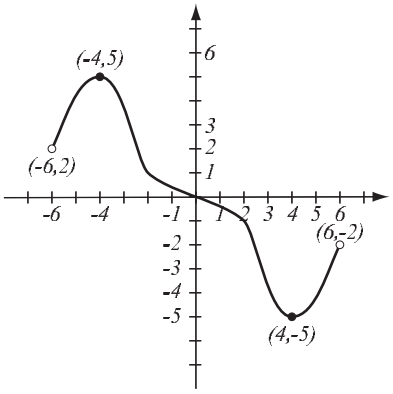
a) Find $f(4)$ and $f(6)$.

b) What is the range of f?

c) Is the function odd, even or neither? How can you tell?

d) Where does f have a local maximum? What is its value?

e) What are the solutions of the equation $f(x) = 3$?



6. (6pts) Solve the inequality. Draw the solution and write it in interval form.

$|x + 4| \geq 5$

7. (12pts) The quadratic function $f(x) = 4x^2 - 8x - 21$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

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8. (14pts) The polynomial $f(x) = (x-3)^2(x+4)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

9. (7pts) Simplify and write the answer so all exponents are positive:

$$\frac{(3x)^3(x^2y^{-1})^5}{18x^2y^{-4}} =$$

10. (8pts) Simplify.

$$\frac{2x-1}{x^2+x-42} + \frac{x}{x^2-36} =$$

11. (5pts) Let $f(x) = 8x^3 - 5$. Find $f^{-1}(x)$.

Solve the equations.

12. (8pts) $3^{2x+5} = 7^{4x-1}$

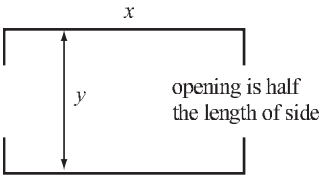
13. (8pts) $x = 3 + \sqrt{-4x + 12}$

14. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log \dfrac{100x^4}{y^3} =$

15. (14pts) Alison and Mitch bicycle along the same road. It takes Mitch 30 minutes to travel the road. Alison leaves 5 minutes after Mitch, but gets to the end of the road at the same time as Mitch because she travels 2.5 mph faster than Mitch.
a) What are the speeds of the cyclists?
b) How long is the road?

16. (14pts) A trucking company wishes to build a service garage for trucks that has openings on two sides that are half the length of the sides (see picture). They have enough money for 1800 feet of walls and wish to maximize the area of the service garage.
a) Express the area of the garage as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the garage that has the biggest possible area?



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