College Algebra — Final Exam
MAT 140, Fall 2014 — D. Ivanšić

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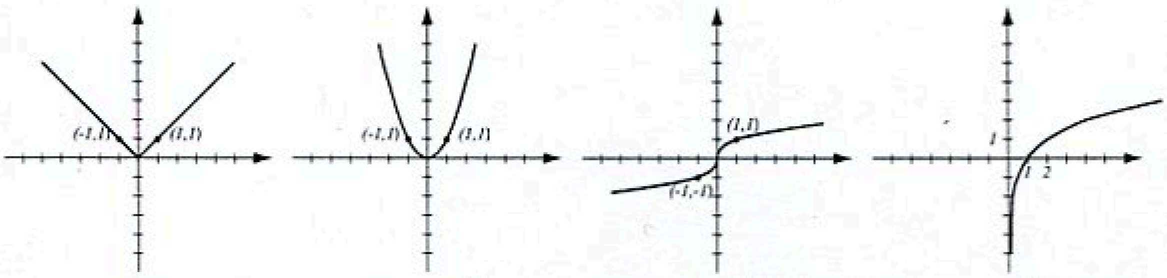
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



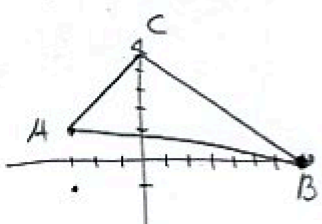
$y = |x|$ $\quad$ $y = x^2$ $\quad$ $y = \sqrt[3]{x}$ $\quad$ $y = \log_a x$

2. (5pts) Find the equation of the line (in form $y = mx + b$) that passes through $(3, 1)$ and $(-2, 5)$.

$m = \frac{5-1}{-2-3} = \frac{4}{-5} = -\frac{4}{5}$ $\quad$ $y - 1 = -\frac{4}{5}(x-3)$ $\quad$ $y = -\frac{4}{5}x + \frac{17}{5}$

$y = -\frac{4}{5}x + \frac{12}{5} + 1$

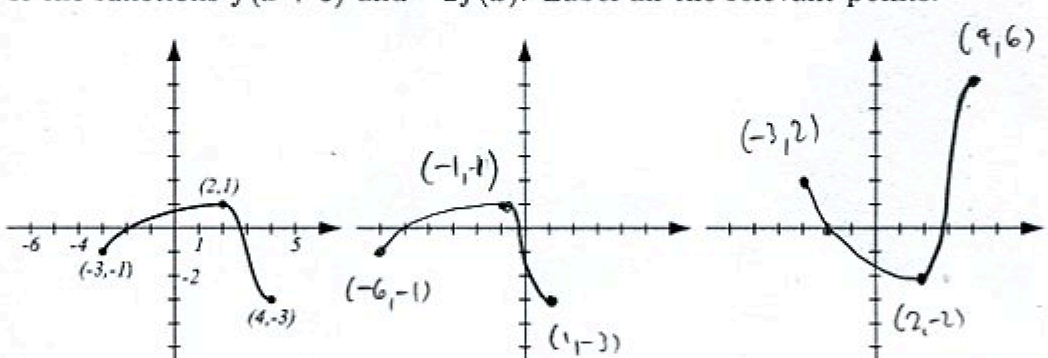
3. (6pts) Is the triangle with vertices $A = (-3, 1)$, $B = (7, 0)$ and $C = (0, 5)$ a right triangle? Use either the distance formula or slopes of perpendicular lines to find out.



line	slope
AB	$\frac{0-1}{7-(-3)} = -\frac{1}{10}$
AC	$\frac{5-1}{0-(-3)} = \frac{4}{3}$
BC	$\frac{5-0}{0-7} = -\frac{5}{7}$

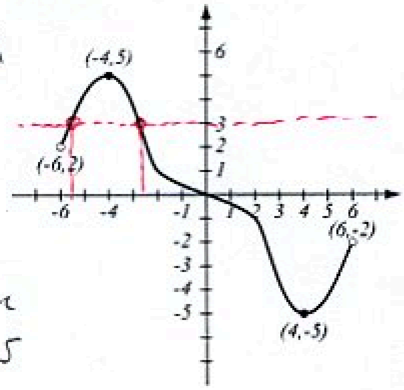
None of these are opposite reciprocal of each other, so no two are perpendicular.

4. (8pts) The graph of the function f is given below. On separate graphs, sketch the graphs of the functions $f(x+3)$ and $-2f(x)$. Label all the relevant points.



shift left 5 $\quad$ stretch vertically, factor 2
reflect in x-axis

5. (10pts) Use the graph of the function f at right to answer the following questions.



a) Find $f(4)$ and $f(6)$. $f(4) = -5$, $f(6) =$ not defined

b) What is the range of f? $[-5, 5]$

c) Is the function odd, even or neither? How can you tell? odd - graph is symm. about the origin

d) Where does f have a local maximum? What is its value? local max at $x=-4$ with value $f(-4)=5$

e) What are the solutions of the equation $f(x) = 3$?

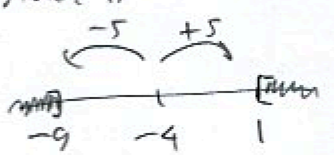
$x = -5.5, -2.75$

6. (6pts) Solve the inequality. Draw the solution and write it in interval form.

$|x + 4| \geq 5$

$|x-(-4)| \geq 5$

distance from x to $-4 \geq 5$ $\quad (-\infty, -9] \cup [1, \infty)$



7. (12pts) The quadratic function $f(x) = 4x^2 - 8x - 21$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

b) $h = -\frac{b}{2a} = -\frac{-8}{2 \cdot 4} = 1$

$k = f(1) = 4 - 8 - 21 = -25$

a) y-int: $f(0) = -21$

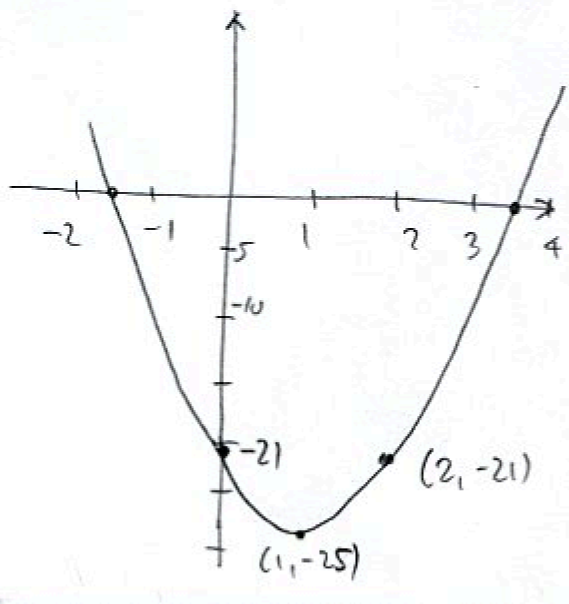
x-int:

$4x^2 - 8x - 21 = 0$

$x = \dfrac{-(-8) \pm \sqrt{(-8)^2 - 4 \cdot 4 \cdot (-21)}}{2 \cdot 4}$

$= \dfrac{8 \pm \sqrt{400}}{8} = \dfrac{8 \pm 20}{8} = -\frac{12}{8}, \frac{28}{8}$

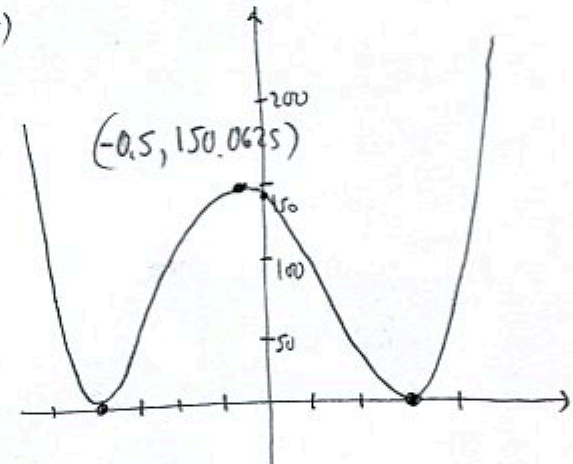
$= -\frac{3}{2}, \frac{7}{2}$



8. (14pts) The polynomial $f(x) = (x-3)^2(x+4)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

a) behaves like x^4 =

c)



b)

zero	multiplicity
$x=3$	2
$x=-4$	2

y-int:
$f(0) = (-3)^2 \cdot 4^2$
$= 144$

d) Turning points:
$f(-4) = 0$ is a local min
$f(-\frac{1}{2}) = 150.0625$ is a local max
$f(3) = 0$ is a local min

9. (7pts) Simplify and write the answer so all exponents are positive:

$$\frac{(3x)^3(x^2y^{-1})^5}{18x^2y^{-4}} = \frac{\overset{3}{\cancel{27}}x^3x^{10}y^{-5}}{\underset{2}{\cancel{18}}x^2y^{-4}} = \frac{3x^{13}y^{-5}}{2x^2y^{-4}} = \frac{3x^{11}y^{-1}}{2} = \frac{3x^4}{2y}$$

10. (8pts) Simplify.

$$\frac{2x-1}{x^2+x-42} + \frac{x}{x^2-36} = \frac{2x-1}{(x+7)(x-6)} + \frac{x}{(x-6)(x+6)}$$

$$= \frac{(2x-1)\cdot(x+6) + x(x+7)}{(x+7)(x-6)(x+6)}$$

$$= \frac{2x^2+11x-6+x^2+7x}{(x+7)(x-6)(x+6)}$$

prod = -2 no such integers
sum = 6
doesn't factor

$$= \frac{3x^2+18x-6}{(x+7)(x-6)(x+6)} = \frac{3(x^2+6x-2)}{(x+7)(x-6)(x+6)}$$

11. (5pts) Let $f(x) = 8x^3 - 5$. Find $f^{-1}(x)$.

$y = 8x^3 - 5$

$y + 5 = 8x^3$

$x^3 = \frac{y+5}{8}$

$x = \sqrt[3]{\frac{y+5}{8}}$

$f^{-1}(y) = \sqrt[3]{\frac{y+5}{8}}$

Solve the equations.

12. (8pts) $3^{2x+5} = 7^{4x-1}$ $\quad | \ln$

$\ln 3^{2x+5} = \ln 7^{4x-1}$

$(2x+5)\ln 3 = (4x-1)\ln 7$

$2x\ln 3 + 5\ln 3 = 4x\ln 7 - \ln 7$

$2x\ln 3 - 4x\ln 7 = -5\ln 3 - \ln 7$

$x(2\ln 3 - 4\ln 7) = -5\ln 3 - \ln 7$

$x = \frac{-5\ln 3 - \ln 7}{2\ln 3 - 4\ln 7} \cdot \frac{(-1)}{(-1)}$

$= \frac{5\ln 3 + \ln 7}{4\ln 7 - 2\ln 3} = 1{,}331618$

13. (8pts) $x = 3 + \sqrt{-4x + 12}$

$x - 3 = \sqrt{-4x+12} \quad |^2$

$x^2 - 6x + 9 = -4x + 12$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

Check: $3 \stackrel{?}{=} 3 + \sqrt{-4 \cdot 3 + 12}$ ✓

$-1 = 3 + \sqrt{4+12}$ no

Only $x = 3$ is the solution.

14. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log \frac{100x^4}{y^3} = \log 100 + \log x^4 - \log y^3 = 2 + 4\log x - 3\log y$

15. (14pts) Alison and Mitch bicycle along the same road. It takes Mitch 30 minutes to travel the road. Alison leaves 5 minutes after Mitch, but gets to the end of the road at the same time as Mitch because she travels 2.5 mph faster than Mitch.
a) What are the speeds of the cyclists?
b) How long is the road?

Mitch $d, r, 30 \text{ min} = \frac{1}{2} \text{ hr}$

Alison $d, r+2.5, 25 \text{ min} = \frac{5}{12} \text{ hr}$

same $d = r \cdot \frac{1}{2}$

$d = (r+2.5) \cdot \frac{5}{12}$

$r \cdot \frac{1}{2} = \frac{5}{12}(r+2.5) \quad \Big| \cdot 12$

$6r = 5(r+2.5)$

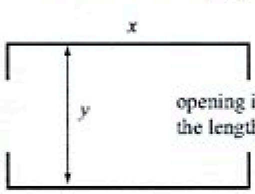
$6r = 5r + 12.5$

$r = 12.5$ mph

a) Mitch rides at 12.5 mph
Alison rides at 15 mph

b) $d = 12.5 \cdot \frac{1}{2} = 6.25$ miles

16. (14pts) A trucking company wishes to build a service garage for trucks that has openings on two sides that are half the length of the sides (see picture). They have enough money for 1800 feet of walls and wish to maximize the area of the service garage.
a) Express the area of the garage as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the garage that has the biggest possible area?



a) $2x + 2 \cdot \frac{1}{2} y = 1800$

$2x + y = 1800$

$y = 1800 - 2x$

$A = xy = x(1800 - 2x)$

$= -2x^2 + 1800x$

$A(x) = -2x^2 + 1800x$

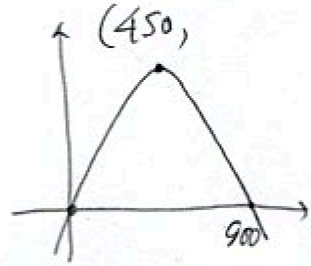
Domain:
Must have
$x \geq 0$
$y \geq 0$
$1800 - 2x \geq 0$

$2x \leq 1800$

$x \leq 900$

$[0, 900]$

b)



$h = -\frac{b}{2a} = -\frac{1800}{2 \cdot (-2)} = 450$

$A(450) = 450 \cdot 900 = 405000 \text{ ft}^2$

Dimensions of biggest rectangle:

450×900

$1800 - 2 \cdot 450$

17. (12pts) The population of Orlando, FL was 128,000 in 1980 and 238,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 1980. Graph it on paper.
b) Find the predicted population in the year 2015.

a) $P(t) = 128\, e^{kt}$ (in thousands)

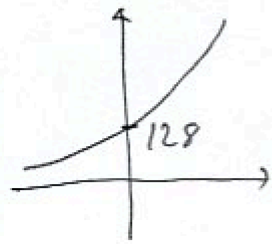
$238 = 128\, e^{k \cdot 30}$

$\frac{238}{128} = e^{k \cdot 30} \quad | \ln$

$\ln \frac{238}{128} = k \cdot 30$

$k = \frac{\ln \frac{238}{128}}{30} = 0.0206747$

$P(t) = 128 e^{0.0206747t}$



b) $P(\underset{2015-1980}{\underset{\uparrow}{35}}) = 128\, e^{0.0206747 \cdot 35}$

$\doteq 263.919485$

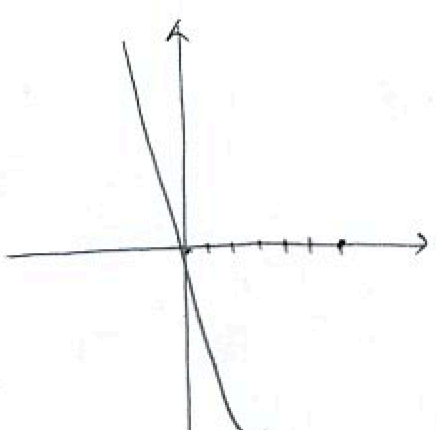
About 263,919 people

Bonus (10pts) Let $f(x) = x^2 - 6x$, considered for $x \leq 3$.
a) Sketch the graph of f and verify that the function is one-to-one.
b) Find the formula for the inverse of this function.

a) x-int:

$x^2 - 6x = 0$

$x(x-6) = 0 \quad x = 0, 6$



Graph is part of parabola $y = x^2 - 6x$ for $x \leq 3$.
This part passes the horizontal line test.

b) $y = x^2 - 6x$

$x^2 - 6x + y = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot y}}{2}$

$= \frac{6 \pm \sqrt{36 - 4y}}{2}$

$= \frac{6 \pm \sqrt{4(9-y)}}{2} = \frac{6 \pm 2\sqrt{9-y}}{2}$

$= 3 \pm \sqrt{9-y}$

since $x \leq 3$, $x = 3 - \sqrt{9-y}$

$f^{-1}(y) = 3 - \sqrt{9-y}$