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| **College Algebra — Exam 4** |
| **MAT 140, Fall 2014 — D. Ivanšić** |

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*Show all your work!*

**1.** (8pts) Evaluate without using the calculator:

$\log_3 81 = 4$ $\qquad$ $\log_5 \frac{1}{125} = -3$ $\qquad$ $\log_a \sqrt[7]{a^3} = \frac{3}{7}$ $\qquad$ $\log_{\sqrt{b}} b^3 = 6$

$3^? = 81$ $\qquad$ $5^? = \frac{1}{125} = \frac{1}{5^3}$ $\qquad$ $a^? = \sqrt[7]{a^3} = a^{3/7}$ $\qquad$ $(\sqrt{b})^? = b^3$

**2.** (4pts) Use the change-of-base formula and your calculator to find $\log_7 17$ with accuracy 6 decimal places. Show how you obtained your number.

$$\log_7 17 = \frac{\log 17}{\log 7} = 1.455984$$

**3.** (5pts) If $\log_a 12 = c$ and $\log_a 5 = d$, express in terms of $c$ and $d$:

$$\log_a 60 = \log_a (5 \cdot 12) = \log_a 5 + \log_a 12 = d + c$$

$$\log_a \frac{144}{125} = \log_a \frac{12^2}{5^3} = \log_a 12^2 - \log_a 5^3 = 2\log_a 12 - 3\log_a 5 = 2c - 3d$$

**4.** (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_7 \frac{49x^3}{\sqrt[3]{y^8}} = \log_7 49 + \log_7 x^3 - \log_7 y^{\frac{8}{3}} = 2 + 3\log_7 x - \frac{8}{3}\log_7 y$$

**5.** (12pts) Write as a single logarithm. Simplify if possible.

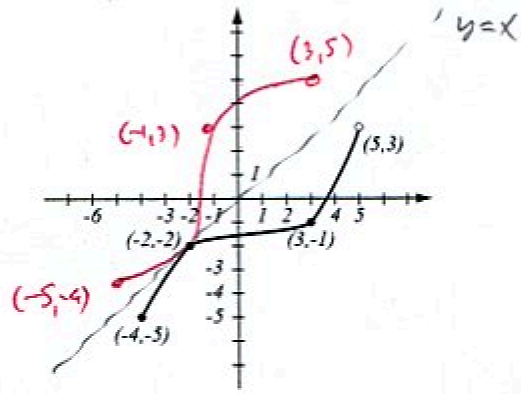
$$2\log(x^3y^4) - 5\log(x^2y^3) = \log (x^3y^4)^2 - \log(x^2y^3)^5 = \log(x^6y^8) - \log(x^{10}y^{15})$$

$$= \log \frac{x^6y^8}{x^{10}y^{15}} = \log x^{-4}y^{-7} = \log \frac{1}{x^4y^7}$$

$$3\ln(x^2+7x-18) - 2\ln(x+9) - 4\ln(x-2) = \ln (\underbrace{x^2+7x-18}_{(x+9)(x-2)})^3 - \ln(x+9)^2 - \ln(x-2)^4$$

$$= \ln \frac{((x+9)(x-2))^3}{(x+9)^2(x-2)^4} = \ln \frac{(x+9)^3(x-2)^3}{(x+9)^2(x-2)^4} = \ln \frac{x+9}{x-2}$$

**6.** (6pts) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points.




a) yes, it passes the horizontal line test

**7.** (9pts) Let $f(x) = \dfrac{3x+1}{4x-1}$, $x \geq 0$.
a) Find the formula for $f^{-1}$.
b) Find the range of $f$.

$$f^{-1}(y) = \frac{y+1}{4y+3}$$

$$y = \frac{3x+1}{4x-1}$$

$$(4x-1)y = 3x+1$$

$$4xy - y = 3x+1 \quad \Big| -3x+y$$

$$4xy - 3x = y+1$$

$$x(4y-3) = y+1$$

$$x = \frac{y+1}{4y-3}$$

range of $f$ = domain of $f^{-1}$

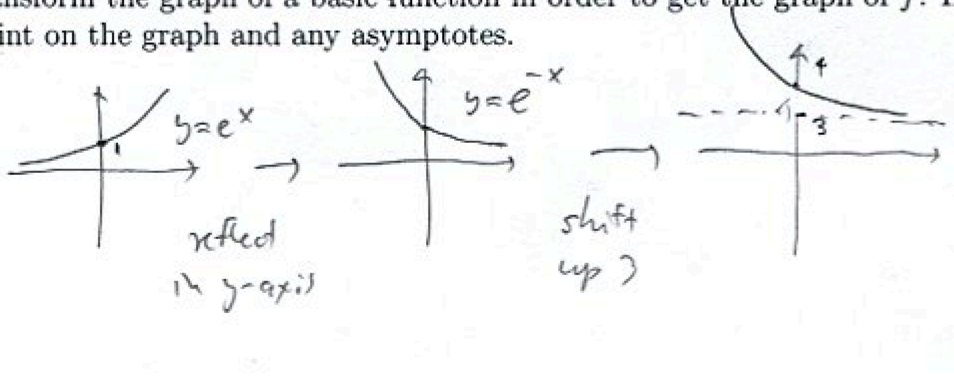
can't have $4y-3=0$

$$4y = 3$$

$$y = \frac{3}{4}$$

Range of $f$: $(-\infty, \frac{3}{4}) \cup (\frac{3}{4}, \infty)$

**8.** (6pts) Using transformations, draw the graph of $f(x) = e^{-x} + 3$. Explain how you transform the graph of a basic function in order to get the graph of $f$. Indicate at least one point on the graph and any asymptotes.



**9.** (6pts) Find the domain of the function $f(x) = \log_2(4x+5) + \log_3(2-7x)$ and write it in interval notation.

Must have: $4x+5>0$ and $2-7x>0$ $\quad -\frac{5}{4}<x<\frac{2}{7}$

$4x>-5 \qquad 2>7x \qquad \left(-\frac{5}{4}, \frac{2}{7}\right)$ is domain

$x>-\frac{5}{4} \qquad \frac{2}{7}>x$

**10.** (8pts) How much should you invest in an account bearing 4.02%, compounded quarterly, if you wish to have \$10,000 in five years?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$10000 = P\left(1+\frac{0.0402}{4}\right)^{4 \cdot 5}$

$10000 = P \cdot 1.221399$

$P = \frac{10000}{1.22..}$

$P = \$8187.33$

Solve the equations.

**11.** (8pts) $2^{x+1} = 3^{1-x}$ $\quad | \ln$

$\ln 2^{x+1} = \ln 3^{1-x}$

$(x+1)\ln 2 = (1-x)\ln 3$

$x\ln 2 + \ln 2 = \ln 3 - x\ln 3$

$x\ln 2 + x\ln 3 = \ln 3 - \ln 2$

$x(\ln 2 + \ln 3) = \ln 3 - \ln 2$

$x = \frac{\ln 3 - \ln 2}{\ln 2 + \ln 3}$

**12.** (10pts) $\log_3(x-2) + \log_3(x+6) = 2$

$\log_3((x-2)(x+6)) = 2 \quad 3^{\frown}$

$3^{\log_3 (x-2)(x+6)} = 3^2$

$x^2+4x-12 = 9$

$x^2+4x-21=0$

$(x+7)(x-3)=0$

$x=-7$ or $x=3$

$\log_3(-7-2)+\log_3(-7+6)$ has neg. under ln

$\log_3(3-2)+\log_3(3+6)$ is ok

$x=3$ is the solution

**13.** (12pts) The population of Orlando, FL was 128,000 in 1980 and 238,000 in 2010. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find $k$ and write the function that describes the population at time $t$ years since 1980. Graph it on paper.
b) Find the predicted population in the year 2015.

a) $P_0 = 128 \quad P(t) = 128 e^{kt}$

$P(30) = 238$

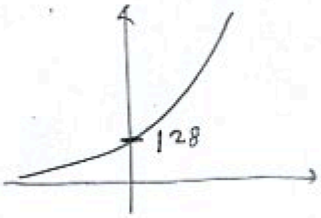
$128 e^{k \cdot 30} = 238$

$e^{k \cdot 30} = \frac{238}{128} \quad | \ln$

$k \cdot 30 = \ln \frac{238}{128}$

$k = \frac{\ln \frac{238}{128}}{30} = 0.0206747$

$P(t) = 128 e^{0.0206747 t}$




b) Pop. in 2015 is

$P(35) = 128 e^{0.0206747 \cdot 35}$

$= 263.919485$

About 263,919 people

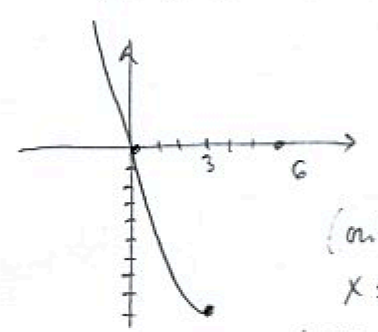
**Bonus** (10pts) Let $f(x) = x^2 - 6x$, considered for $x \leq 3$.
a) Sketch the graph of $f$ and verify that the function is one-to-one.
b) Find the formula for the inverse of this function.

a) $x^2 - 6x = x(x-6)$

x-int: $0, 6$

vertex: $h = -\frac{b}{2a} = -\frac{-6}{2 \cdot 1} = 3$

$k = 9 - 18 = -9$




(only part of parabola for $x \leq 3$ is the graph,
- passes horizontal line test

b) $y = x^2 - 6x$

$x^2 - 6x - y = 0$

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot (-y)}}{2 \cdot 1} = \frac{6 \pm \sqrt{36 + 4y}}{2}$

$= \frac{6 \pm \sqrt{4(9+y)}}{2} = \frac{6 \pm 2\sqrt{9+y}}{2} = 3 \pm \sqrt{9+y}$

since $x \leq 3$ we have to take the solution with a $-$: $x = 3 - \sqrt{9+y}$

$f^{-1}(y) = 3 - \sqrt{9+y}$