College Algebra — Exam 3
MAT 140, Fall 2014 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (4pts) $3i(5 - i) + 3 - 2i =$ $15i - 3\underbrace{i^2}_{-1} + 3 - 2i = 13i + 3 + 3 = 6 + 13i$

2. (6pts) $\frac{3+4i}{4-3i} =$ $\frac{3+4i}{4-3i} \cdot \frac{4+3i}{4+3i} = \frac{12 + 9i + 16i + 12\overset{-1}{i^2}}{4^2 - (3i)^2} = \frac{25i}{16+9} = \frac{25i}{25} = i$

3. (4pts) Simplify and justify your answer.

$i^{155} =$ $i^{152} i^3 = 1 \cdot i \cdot i \cdot i = -i$

multiple of 4

$152 = 4 \cdot 38$

4. (6pts) Solve the equation by completing the square.

$x^2 - 12x + 41 = 0$ $\quad | + 6^2$

$x^2 - 12x + 6^2 = 6^2 - 41$

$(x-6)^2 = -5$

$x - 6 = \pm\sqrt{5}i$

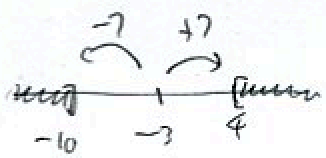
$x = 6 \pm \sqrt{5}i$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x + 3| \geq 7$

$|x - (-3)| \geq 7$

distance from x to $-3 \geq 7$



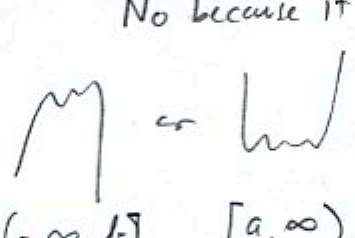
$(-\infty, -10] \cup [4, \infty)$

6. (6pts) Let $f(x)$ be some polynomial of degree 4.

a) State the maximum number of x-intercepts of the graph of f. 4

b) State the maximum number of turning points on the graph of f. 3

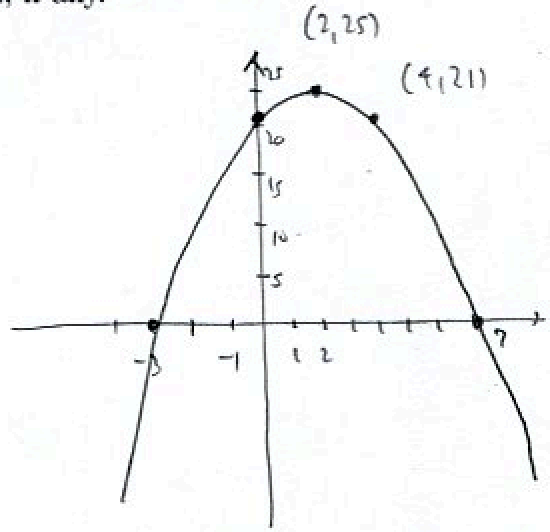
c) Can the range of f be $(-\infty, \infty)$? Why or why not? No because it looks like



Neither has range $(-\infty, \infty)$

range: $(-\infty, b]$ $\quad [a, \infty)$

7. (12pts) The quadratic function $f(x) = -x^2 + 4x + 21$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.



a) y-int: $f(0) = 21$

x-int $-x^2 + 4x + 21 = 0$

$x^2 - 4x - 21 = 0$

$(x-7)(x+3) = 0$

$x = 7, -3$

b) $h = -\frac{b}{2a} = -\frac{4}{2\cdot(-1)} = 2$

$k = f(2) = -4 + 8 + 21 = 25$

Solve the equations:

8. (8pts) $\frac{x+5}{x+3} + \frac{8}{x^2+2x-3} = \frac{4}{x-1}$ $\Big| \cdot LCD$ $(x+3)(x-1)$

$(x+3)(x-1)$

$(x+5)(x-1) + 8 = 4(x+3)$

$x^2 + 4x - 5 + 8 = 4x + 12 \quad \Big| -4x - 12$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$

$x = -3$ gives 0 in denom, so the only solution is $\boxed{x = 3}$

9. (8pts) $\sqrt{4t+5} + \sqrt{t+5} = 3$

$\sqrt{4t+5} = 3 - \sqrt{t+5} \quad \Big|^2$

$4t + 5 = 9 - 2\cdot 3\sqrt{t+5} + t + 5 \quad \Big| -t - 14$

$3t - 9 = -6\sqrt{t+5} \quad \Big| \div 3$

$t - 3 = -2\sqrt{t+5} \quad \Big|^2$

$t^2 - 6t + 9 = 4(t+5) \quad \Big| -4t - 20$

$t^2 - 10t - 11 = 0$

$(t-11)(t+1) = 0$

$t = 11, -1$

check: $t = 11$ $\sqrt{49} + \sqrt{16} \stackrel{?}{=} 3$ no

$t = -1$ $\sqrt{1} + \sqrt{4} = 3$ yes

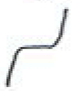
Solution: $\boxed{t = -1}$

10. (14pts) The polynomial $f(x) = (x+5)(x-2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

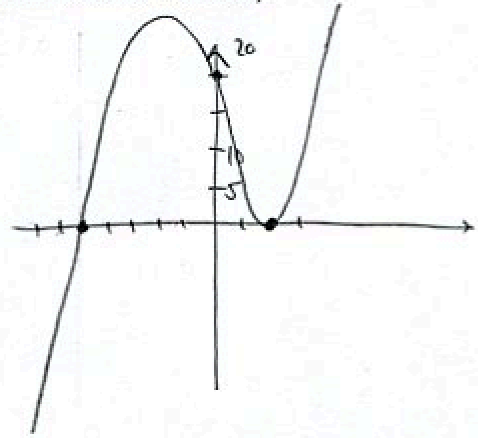
a) $(x+5)(x-2)^2$

leading $x \cdot x^2 = x^3$

like x^3



c)



b)

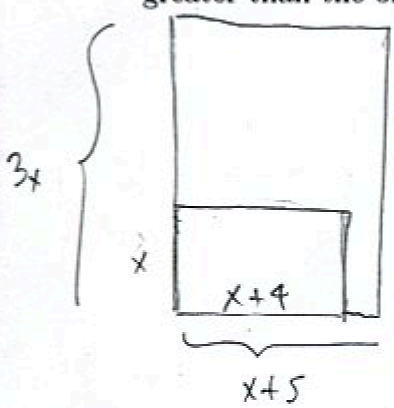
zero	-5	2
mult	1	2

y-int: $f(0) = 5 \cdot (-2)^2 = 20$

d) $f(2) = 0$ is a local minimum

$f(-2.666668) = 50.814815$ is a local max

11. (12pts) One side of a rectangle is 4 meters longer than the other. If the shorter side is tripled and the longer side extended by 1 meter, the resulting rectangle has area 63 m^2 greater than the original one. What are the dimensions of the original rectangle?



small rectangle area + 63 = large rectangle area

$x(x+4) + 63 = 3x(x+5)$

$x^2 + 4x + 63 = 3x^2 + 15x$

$2x^2 + 11x - 63 = 0$

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \cdot 2 \cdot (-63)}}{2 \cdot 2} = \frac{-11 \pm \sqrt{\overbrace{121 + 504}^{625}}}{4} = \frac{-11 \pm 25}{4}$$

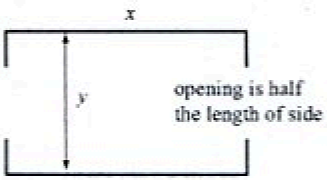
$$= \frac{14}{4}, -\frac{36}{4} = \frac{7}{2}, -9$$ x can't be negative

$x = \frac{7}{2} = 3.5$, rectangle is 3.5×7.5

12. (14pts) A trucking company wishes to build a service garage for trucks that has openings on two sides that are half the length of the sides (see picture). They have enough money for 1200 feet of walls and wish to maximize the area of the service garage.

a) Express the area of the garage as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the garage that has the biggest possible area?



$2x + 2\cdot\frac{y}{2} = 1200$

$2x + y = 1200$

$y = 1200 - 2x$

$A = xy = x(1200-2x)$

$A(x) = -2x^2 + 1200x$

Domain $x \geq 0$, $y \geq 0$

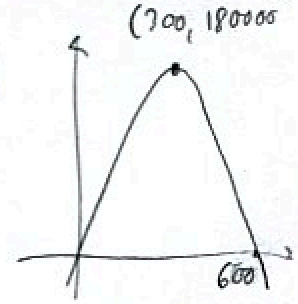
$1200 - 2x \geq 0$

$1200 \geq 2x$

$x \leq 600$

$[0, 600]$

b)



vertex is $h = -\frac{1200}{2\cdot(-2)} = 300$

$k = 300\cdot(1200 - 2\cdot 300) = 180000\ ft^2$

Dimensions of garage

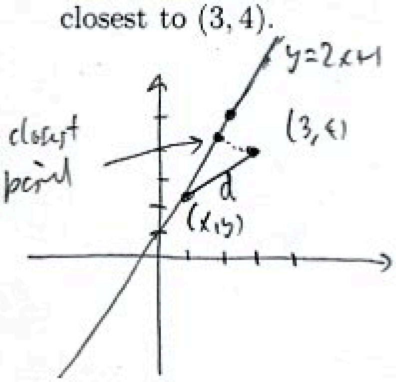
$x = 300 \quad y = 1200 - 2\cdot 300 = 600$

300×600

Bonus. (10pts) Follow the steps below to find the point on the line $y = 2x + 1$ that is closest to the point $(3, 4)$. (Sketch the line and the point.)

a) Write the function that represents the square of the distance (avoids a square root) from a point (x, y) on the line to the point $(3, 4)$ and express it in terms of x.

b) Graph the function in order to find the minimum and state the point on the line that is closest to $(3, 4)$.



$d^2 = (x-3)^2 + (y-4)^2 = (x-3)^2 + (2x+1-4)^2$

$y = 2x+1$ since (x,y) is on line

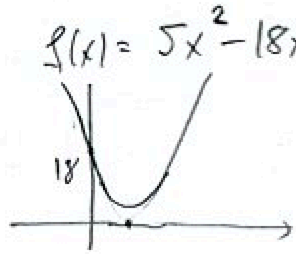
$= (x-3)^2 + (2x-3)^2 = x^2 - 6x + 9 + 4x^2 - 12x + 9$

$f(x) = 5x^2 - 18x + 18$

vertex: $h = -\frac{b}{2a} = -\frac{-18}{2\cdot 5} = \frac{18}{10} = \frac{9}{5} = 1.8$

$k = 5\left(\frac{9}{5}\right)^2 - 18\cdot\frac{9}{5} + 18 = \frac{81}{5} - \frac{162}{5} + \frac{90}{5}$

$= \frac{9}{5}$



Minimal distance of $\frac{9}{5}$ is achieved at point $\left(\frac{9}{5}, \frac{23}{5}\right)$ $\quad 2\cdot\frac{9}{5}+1$