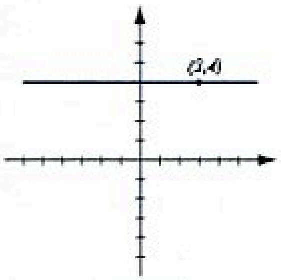
College Algebra — Exam 2
MAT 140, Fall 2014 — D. Ivanšić

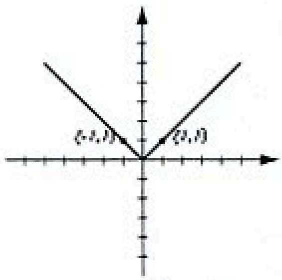
Name: Saul Ocean

Show all your work!

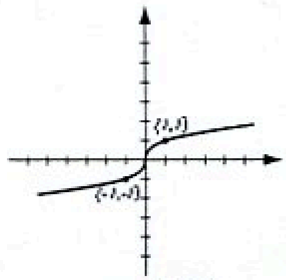
1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



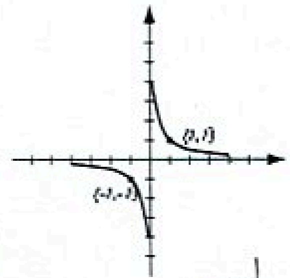
$y = 4$



$y = |x|$



$y = \sqrt[3]{x}$



$y = \frac{1}{x}$

2. (10pts) Find the equation of the line (in form $y = mx + b$) that passes through $(1, 2)$ and is perpendicular to the line $2x + 5y = 3$. Draw both lines.

$2x + 5y = 3$

$5y = -2x + 3$

$y = -\frac{2}{5}x + \frac{3}{5}$

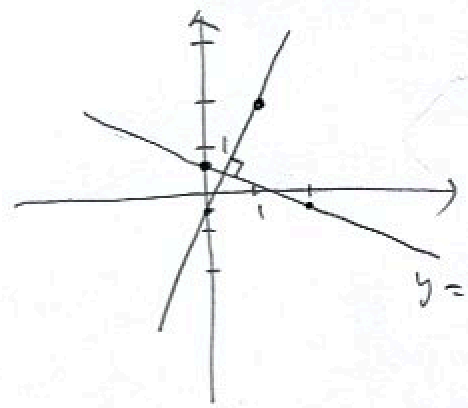
slope of perp. line is

$-\frac{1}{-\frac{2}{5}} = \frac{5}{2}$

$y - 2 = \frac{5}{2}(x - 1)$

$y - 2 = \frac{5}{2}x - \frac{5}{2}$

$y = \frac{5}{2}x - \frac{1}{2}$



$y = -\frac{2}{5}x + \frac{3}{5}$

3. (5pts) Solve the inequality and write your solution in interval notation.

$3 \le 3x - 1 < 7$ $\quad |+1$

$4 \le 3x < 8$ $\quad |\div 3$

$\frac{4}{3} \le x < \frac{8}{3}$

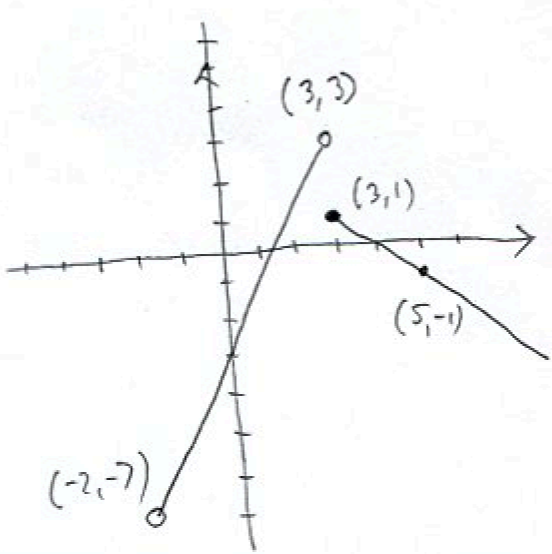
$\left[\frac{4}{3}, \frac{8}{3}\right)$

4. (8pts) Sketch the graph of the piecewise-defined function:

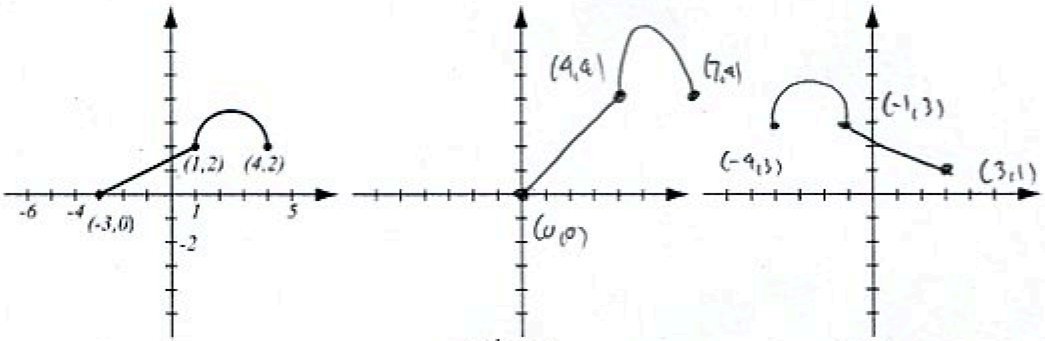
$$f(x) = \begin{cases} 2x - 3, & \text{if } -2 < x < 3 \\ 4 - x, & \text{if } x \ge 3 \end{cases}$$

x	$2x-3$
-2	-7
3	3

x	$4-x$
3	1
5	-1



5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $2f(x-3)$ and $f(-x)+1$ and label all the relevant points.



right 3
stretch vert. by factor 2

reflect in y-axis
shift up 1

6. (14pts) Let $f(x) = \sqrt{2x+1}$, $g(x) = x^2 + 3$.

Find the following (simplify where possible):

$(f+g)(0) = f(0) + g(0)$
$= \sqrt{2 \cdot 0 + 1} + 0^2 + 3$
$= 4$

$(fg)(x) = f(x)g(x)$
$= \sqrt{2x+1} \cdot (x^2+3)$
$= (x^2+3)\sqrt{2x+1}$

$(f \circ g)(-1) = f(g(-1))$
$= f((-1)^2+3) = f(4)$
$= \sqrt{2 \cdot 4 + 1} = 3$

$(g \circ f)(x) = g(f(x))$
$= g(\sqrt{2x+1})$
$= (\sqrt{2x+1})^2 + 3 = 2x+1+3$
$= 2x+4$

The domain of f in interval notation

Must have $2x+1 \geq 0$
$2x \geq -1$
$x \geq -\frac{1}{2}$

$[-\frac{1}{2}, \infty)$

7. (4pts) Consider the function $h(x) = \sqrt[3]{x^2 - 2x + 4}$. Find functions f and g, neither of which is the "stupid" one, so that $h(x) = f(g(x))$.

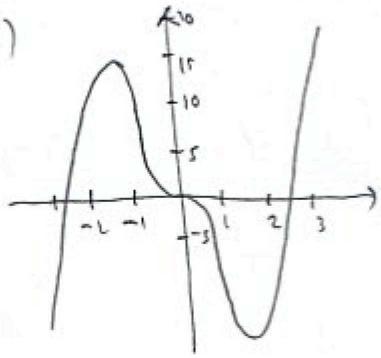
$g(x) = x^2 - 2x + 4$
$f(x) = \sqrt[3]{x}$

or

$g(x) = x^2 - 2x$
$f(x) = \sqrt[3]{x+4}$

8. (17pts) Let $f(x) = x^5 - 6x^3$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate scale on the graph.
b) Determine algebraically whether f is even, odd, or neither. Then state how the graph supports your conclusion.
c) Find the local maxima and minima for this function.
d) State the intervals where the function is increasing and where it is decreasing.

a)



c) $f(-1.897365) = 16.393247$ is a local maximum

$f(1.897365) = 16.393247$ is a local minimum

d) Increasing on

$(-\infty, -1.897365)$ and $(1.897365, \infty)$

decreasing on $(-1.897365, 1.897365)$

b) $f(-x) = (-x)^5 - 6(-x)^3$

$= -x^5 - 6(-x^3)$

$= -x^5 + 6x^3$

$= -(x^5 - 6x^3)$

$= -f(x)$

f is odd, - graph is symmetric about the origin

9. (10pts) Prices for apples at two orchards are: Fufu Farms charges \$25 packing and 67 cents per pound, while Old McDonald's charges \$10 packing and 73 cents per pound. For which quantities of apples is Fufu Farms the better deal?

x = pounds bought

Fufu Farms cost: $25 + 0.67x$

Old McDonald's $10 + 0.73x$

$25 + 0.67x \leq 10 + 0.73x$

$15 \leq 0.06x$

$x \geq \frac{15}{0.06} = 250$

If one buys 250 lbs or more, Fufu Farms is the better deal.

10. (14pts) Alison and Mitch bicycle along the same road. It takes Mitch 30 minutes to travel the road. Alison leaves 6 minutes after Mitch, but gets to the end of the road at the same time as Mitch because she travels 1.5 mph faster than Mitch.
a) What are the speeds of the cyclists?
b) How long is the road?

Mitch: d, r, $\frac{1}{2}$ hr

Alison: d, $r+1.5$, $\frac{24}{60}$ hr

same: $d = r \cdot \frac{1}{2}$, $d = (r+1.5)\cdot\frac{24}{60}$

$\frac{1}{2} r = \frac{24}{60}(r+1.5)$

$\frac{1}{2} r = \frac{2}{5}(r+1.5) \quad | \cdot 10$

$5r = 4(r+1.5)$

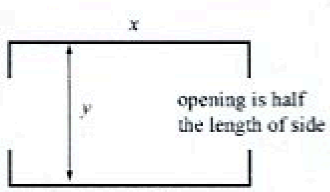
$5r = 4r + 6 \quad | -r$

$r = 6$ mph

a) Mitch rides at 6 mph
Alison at 7.5 mph

b) $d = 6 \cdot \frac{1}{2} = 3$ miles

Bonus. (14pts) A trucking company wishes to build a service garage for trucks that is to have area 6400 square feet, and has openings on two sides that are half the length of the sides (see picture). To minimize cost, the total length of walls has to be as small as possible.
a) Express the total length of walls of the garage as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the garage that has the smallest total wall length?



a) $\ell(x) = 2x + \frac{6400}{x}$

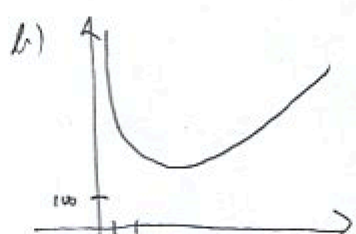
Domain: $x > 0$

$\ell = 2x + \frac{y}{2} + \frac{y}{2} = 2x + y$

$xy = 6400$

$y = \frac{6400}{x}$

b)



Wall length is minimal (226.27417) for

$x = 56.568538 \quad y = 113.137095$

$\frac{6400}{x}$