

1. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

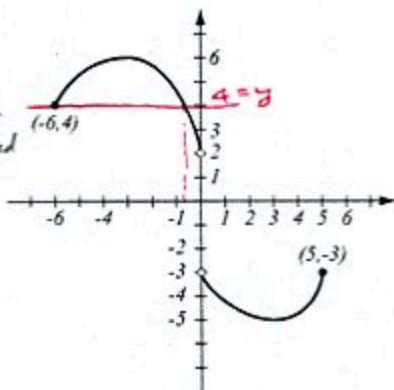
a) Find  $f(3)$  and  $f(0)$ .  $f(3) = -5$   $f(0)$  <sup>not defined</sup>

b) What is the domain of  $f$ ?  $[-6, 0) \cup (0, 5]$

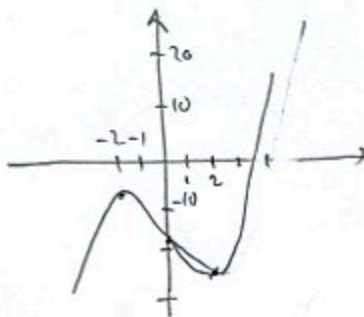
c) What is the range of  $f$ ?  $[-5, -3] \cup (2, 6]$

d) What are the solutions of the equation  $f(x) = 4$ ?

$$x = -6, -0.75$$



2. (10pts) Use your calculator to accurately sketch the graph of  $y = x^3 - 10x - 17$ . Draw the graph here, and indicate units on the axes. Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).



$$y\text{-int} : -17$$

$$x\text{-int} : 3.803837$$

3. (4pts) Convert to scientific notation or a decimal number:

$$0.0000347 = 3.47 \times 10^{-5}$$

$$1.593 \times 10^4 = 15,930$$

Use formulas to expand:

4. (4pts)  $(4x + 5)^2 = (4x)^2 + 2 \cdot 4x \cdot 5 + 5^2 = 16x^2 + 40x + 25$

5. (4pts)  $(2x - u^2)(2x + u^2) = (2x)^2 - (u^2)^2 = 4x^2 - u^4$

6. (6pts) Factor:  $8x^3 - 125 = (2x)^3 - 5^3 = (2x - 5)((2x)^2 + 2x \cdot 5 + 5^2)$   
 $= (2x - 5)(4x^2 + 10x + 25)$

Simplify, showing intermediate steps. Assume variables can be any real numbers.

$$7. (2\text{pts}) \sqrt[3]{108} = \sqrt[3]{27 \cdot 4}$$

$$= 3 \sqrt[3]{4}$$

$$8. (5\text{pts}) \sqrt{125x^7y^4} = \sqrt{25 \cdot 5 \cdot x^6 \cdot x \cdot y^4}$$

$$= 5\sqrt{5} \sqrt{(x^3)^2} \sqrt{(y^2)^2} \sqrt{x}$$

$$= 5|x^3||y^2|\sqrt{5x}$$

9. (8pts) Simplify.

$$\frac{x-5}{3x^2-x-10} - \frac{2x}{x^2+3x-10} = \frac{(x-5)(x+5) - 2x(3x+5)}{(3x+5)(x-2)(x+5)}$$

$\text{prod} = -30$   
 $\text{sum} = -1$   
 $(x+5)(x-2)$   
 $3x^2 - 6x + 5x - 10$   
 $= 3x(x-2) + 5(x-2)$   
 $= (3x+5)(x-2)$

$$= \frac{x^2 - 25 - 6x^2 - 10x}{(3x+5)(x-2)(x+5)} = \frac{-5x^2 - 10x - 25}{(3x+5)(x-2)(x+5)}$$

$$= \frac{-5(x^2 + 2x + 5)}{(3x+5)(x-2)(x+5)}$$

$\text{prod} = 5$   
 $\text{sum} = 2$   
*no sol.*  
*doesn't factor*

10. (8pts) Simplify. Express answers first in terms of positive exponents, then convert to radical notation.

$$\frac{(x^3y^{-\frac{1}{2}})^{\frac{3}{4}}}{(x^{\frac{2}{3}}y^4)^{\frac{1}{4}}} = \frac{(x^3)^{\frac{3}{4}}(y^{-\frac{1}{2}})^{\frac{3}{4}}}{(x^{\frac{2}{3}})^{\frac{1}{4}}(y^4)^{\frac{1}{4}}} = \frac{x^{\frac{9}{4}}y^{-\frac{3}{8}}}{x^{\frac{1}{6}}y^1} = x^{\frac{9}{4}-\frac{1}{6}}y^{-\frac{3}{8}-1} = x^{\frac{27-2}{12}}y^{\frac{-3-8}{8}}$$

$$= x^{\frac{25}{12}}y^{-\frac{11}{8}} = \frac{x^{\frac{25}{12}}}{y^{\frac{11}{8}}} = \frac{\sqrt[12]{x^{25}}}{\sqrt[8]{y^{11}}}$$

11. (6pts) Rationalize the denominator.

$$\frac{4-5\sqrt{3}}{\sqrt{3}+2} \cdot \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{4\sqrt{3}-8-5\sqrt{3}^2+10\sqrt{3}}{\sqrt{3}^2-2^2} = \frac{-14\sqrt{3}-23}{3-4} = 23-14\sqrt{3}$$

$$\frac{-14\sqrt{3}-23}{\underbrace{3-4}_{=-1}} = 23-14\sqrt{3}$$

12. (5pts) Solve the equation for  $t$ .

$$c(a + bt) = d$$

$$ca + cbt = d$$

$$cbt = d - ca$$

$$t = \frac{d - ca}{cb}$$

13. (8pts) Find the domain of the function  $f(x) = \frac{1 + \sqrt{x}}{x^2 + 2x - 8}$  and write it using interval notation.

Can't have:

$$x^2 + 2x - 8 = 0$$

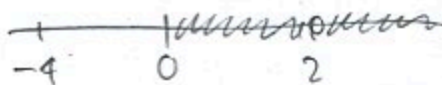
$$(x + 4)(x - 2) = 0$$

$$x = -4, 2$$

Must have

$$x \geq 0$$

$$\text{Domain} = [0, 2) \cup (2, \infty)$$



14. (10pts) Let  $g(x) = (x^2 + 2)\sqrt{3 - x}$ . Find the following (simplify where appropriate).

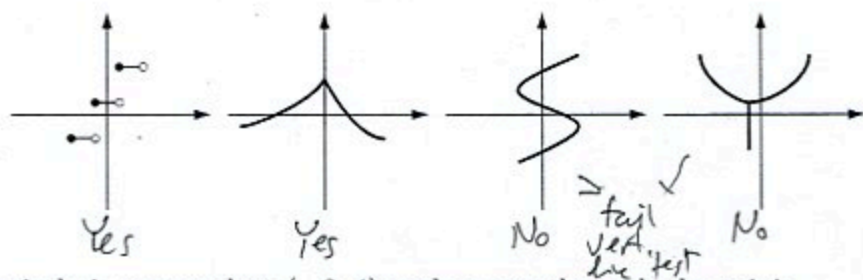
$$\begin{aligned} g(-1) &= ((-1)^2 + 2)\sqrt{3 - (-1)} \\ &= 3\sqrt{4} = 6 \end{aligned}$$

$$g(8) = (64 + 2)\sqrt{-5} \text{ not defined}$$

$$\begin{aligned} g(\sqrt{a}) &= (\sqrt{a}^2 + 2)\sqrt{3 - \sqrt{a}} \\ &= (a + 2)\sqrt{3 - \sqrt{a}} \end{aligned}$$

$$\begin{aligned} g(x-1) &= ((x-1)^2 + 2)\sqrt{3 - (x-1)} \\ &= (x^2 - 2x + 1 + 2)\sqrt{3 - x + 1} \\ &= (x^2 - 2x + 3)\sqrt{4 - x} \end{aligned}$$

15. (4pts) Which of the following graphs are graphs of functions (yes/no)?



16. (8pts) A circle is centered at  $(-3, 4)$  and passes through the origin.

- a) Find the equation of the circle.  
b) Draw the circle in the coordinate plane.

a)  $r =$  distance from  $(-3, 4)$  to  $(0, 0)$

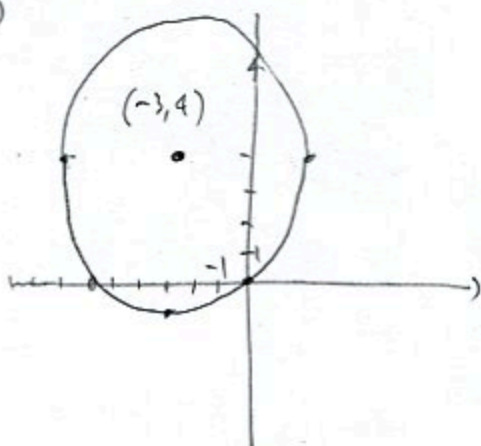
$$r = \sqrt{(0 - (-3))^2 + (0 - 4)^2}$$

$$= \sqrt{9 + 16} = 5$$

Equation of circle:

$$(x - (-3))^2 + (y - 4)^2 = 5^2$$

$$(x + 3)^2 + (y - 4)^2 = 25$$



**Bonus** (10pts) Find points on the  $x$ -axis whose distance to point  $(3, 2)$  is  $\sqrt{29}$ . *Hint: what form do coordinates of points on the  $x$ -axis have?*

A point on the  $x$ -axis has coordinates  $(x, 0)$

$$d = \sqrt{(x - 3)^2 + (0 - 2)^2} = \sqrt{29}$$

$$(x - 3)^2 + 4 = 29$$

$$(x - 3)^2 = 25$$

$$x - 3 = \pm 5$$

$$x = 3 \pm 5 = 8, -2$$

The points are  $(8, 0), (-2, 0)$