

Calculus 1 — Exam 7
MAT 250, Spring 2012 — D. Ivanšić

Name: _____
Show all your work!

1. (10pts) Use the graph of the function to answer the following. Justify your answer if a limit does not exist.

$$\lim_{x \rightarrow 1^-} f(x) =$$

$$\lim_{x \rightarrow 1^+} f(x) =$$

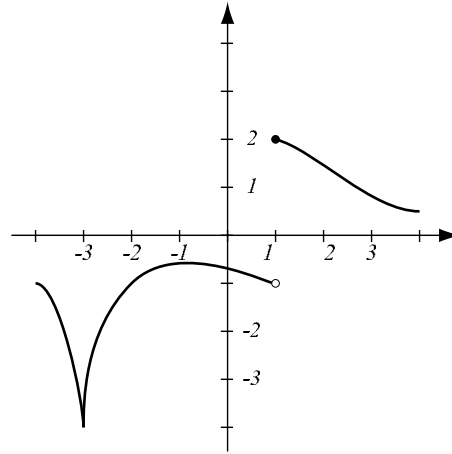
$$\lim_{x \rightarrow 1} f(x) =$$

$$f(1) =$$

$$\lim_{x \rightarrow -3} f(x) =$$

List points where f is not continuous and explain why.

List points where f is not differentiable and explain why.



2. (12pts) Find the following limit a) algebraically and b) using L'Hospital's rule.

$$\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x - 3} =$$

3. (10pts) Let $f(x) = xe^{-x}$. Find the absolute minimum and maximum values of f on the interval $[0, 2]$.

4. (12pts) Use implicit differentiation to find the equation of the tangent line to the curve $3x^2 + 4y^2 + 3xy = 25$ at point $(1, 2)$.

5. (12pts) Researchers have found that the volume of a certain marine organism is linked to its surface area via the formula $V = \frac{A^{\frac{3}{2}}}{12}$.

a) Find the volume of the organism if its surface area is 9cm^2 .

b) Find the ROC of volume with respect to surface area when $A = 9$ (units?).

c) Use b) to estimate the change in volume if surface area increases by 2cm^2 .

d) Use c) to estimate the volume of the organism whose surface area is 11cm^2 and compare to the actual value of 3.0402cm^3 .

6. (24pts) Let $f(x) = \frac{x^2 + 12}{x - 4}$. Draw an accurate graph of f by following the guidelines.

a) Find the intervals of increase and decrease, and local extremes.

b) Find the intervals of concavity and points of inflection.

c) Find $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$.

d) Use information from a)–c) to sketch the graph.

7. (10pts) Use logarithmic differentiation to find the derivative of $y = \left(1 + \frac{1}{x}\right)^x$.

8. (10pts) Let $f(x) = \ln x$.

a) Find the first four derivatives of f .

b) Find the general formula for $f^{(n)}(x)$.

9. (12pts) Among all rectangles of perimeter 20, find the one with the largest area. Show that the area is, indeed, maximal at the point you found.

10. (8pts) Find $f(x)$ if $f'(x) = x^2(x + \sqrt{x})$, if $f(1) = 7$.

11. (8pts) Consider the integral $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin \theta \, d\theta$.

- Use a picture to determine whether this definite integral is positive or negative.
- Evaluate the integral and verify your conclusion from a).

12. (8pts) Use the substitution rule to find the integral:

$$\int_3^{12} \frac{x-1}{\sqrt[3]{x^2-2x+5}} dx =$$

13. (14pts) Luke Skywalker finds himself in a 4 meters wide rectangular garbage compactor containing 30m^3 of water. The side walls are closing in, causing its length to decrease at rate 1 meter per minute and the water level to rise (width stays constant). How fast is the water level increasing when depth is 1.5 meters?

Bonus. (15pts) Let $f(x) = \frac{1}{2}x + \sin x$.

a) On the interval $[0, 4\pi]$, where is the function increasing or decreasing?

b) Show that $\lim_{x \rightarrow \infty} f(x) = \infty$. Use the theorem that rhymes with gentle air movement.