Calculus 1 — Exam 6
MAT 250, Spring 2012 — D. Ivanšić

Name: _____

Show all your work!

Find the following antiderivatives.

1. (3pts) $\int e^{3x+2}\,dx =$

2. (7pts) $\int \frac{x^2-4x}{\sqrt{x}}\,dx =$

3. (4pts) $\int \sec^2(3\theta)\,d\theta =$

4. (16pts) Find $\int_0^3 |x-2|\,dx$ in two ways (they'd better give you the same answer!):
a) Using the "area" interpretation of the integral. Draw a picture.
b) Using the Fundamental Theorem of Calculus (you will have to break it up into two integrals).

5. (6pts) Evaluate: $\sum_{i=3}^{100}(3i-2) =$

Use the substitution rule in the following integrals:

6. (9pts) $\int x^2(x^3-1)^{\frac{3}{2}}\,dx =$

7. (9pts) $\int_e^{e^2} \frac{1}{x\ln x}\,dx =$

8. (9pts) $\int_0^{\frac{\pi}{4}} \frac{\sin\theta}{\cos^2\theta}\,d\theta =$

9. (8pts) The velocity of a vibrating spring is $v(t) = 13 \sin 2t$ (in centimeters per second). Find its position function $s(t)$ if $s(0) = 12$ centimeters.

10. (21pts) The function $f(x) = x^2$, $0 \leq x \leq 2$ is given.
a) Write down the expression that is used to compute R_6. Then compute R_6.
b) Illustrate with a diagram, where appropriate rectangles are clearly visible. What does R_6 represent? Does it over- or underestimate the area under the curve.
c) Using the Fundamental Theorem of Calculus, evaluate $\int_0^2 x^2\,dx$. How far off is R_6?

11. (8pts) Show that $\dfrac{\pi}{12} \leq \displaystyle\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan x \, dx \leq \dfrac{\sqrt{3}\pi}{12}$ **without** evaluating the integral.

Bonus. (10pts) The rate at which water flows into a tank is given by the formula $1 - \frac{1}{2}t$ liters per minute. At time $t = 0$, there were 5 liters of water in the tank.
a) When is the tank filling with water, and when is it draining.?
b) How much water got added (or drained) from the tank from $t = 0$ to $t = 6$?
c) How much water is in the tank when $t = 6$?