Calculus 1 — Exam 5
MAT 250, Spring 2012 — D. Ivanšić

Name: _____

Show all your work!

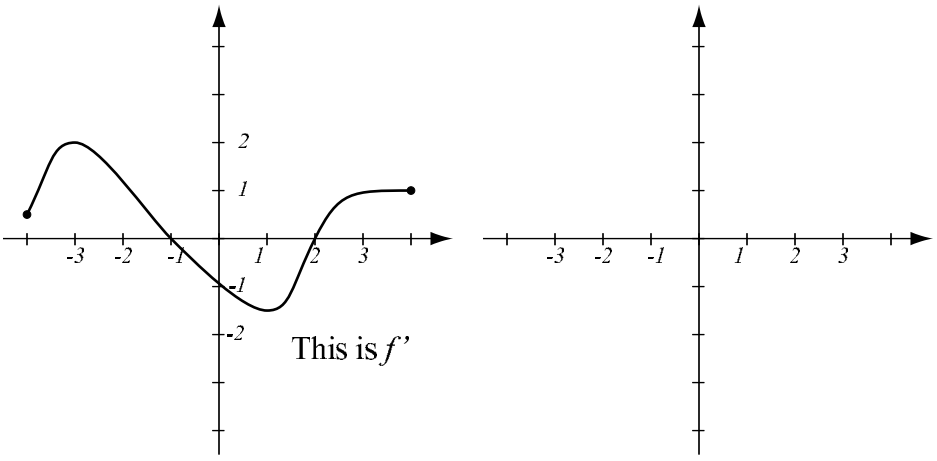
Find the limits. Use L'Hopital's rule where appropriate.

1. (8pts) $\lim_{x\to\infty} \dfrac{7x^2-3x+4}{\sqrt{3x^4-4x^3+5}} =$

2. (8pts) $\lim_{x\to 0} \dfrac{\cos x - 1}{x^2} =$

3. (10pts) $\lim_{x\to 0+} (\cos x)^{\frac{1}{x}} =$

4. (14pts) Let f be continuous on $[-4, 4]$. The graph of its derivative f' is drawn below. Use the graph to answer:
a) What are the intervals of increase and decrease of f? Where does f have a local minimum or maximum?
b) What are the intervals of concavity of f? Where does f have inflection points?
c) Use the information gathered in a) and b) to draw one possible graph of f at right.



5. (18pts) Let $f(x) = \dfrac{\ln x}{x^3}$, $x > 0$.
a) Find the intervals of concavity and points of inflection for f.
b) Find $\lim_{x\to\infty} f(x)$, and use it, along with concavity, to draw the graph of f for $x > 10$. (You don't need to investigate where f is increasing or decreasing, just draw the right tail-end of f.)

6. (26pts) Let $f(x) = \dfrac{x}{x^2+9}$. Draw an accurate graph of f by following the guidelines.

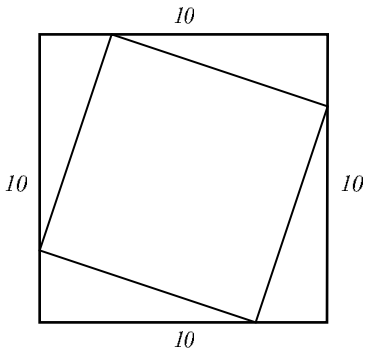
a) Find the intervals of increase and decrease, and local extremes.

b) Find the intervals of concavity and points of inflection.

c) Find $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

d) Use information from a)–d) to sketch the graph.

7. (16pts) A square is inscribed into a larger square with side length 10, as in the picture.
a) Draw two more possibilities for the inscribed square.
b) Find the inscribed square that has the minimal area.



Bonus. (10pts) Show that $\ln x$ grows slower than any root function. That is, show that for any integer $n > 0$, $\lim_{x\to\infty} \dfrac{\ln x}{\sqrt[n]{x}} = 0$.